

# Demographics, Wealth, and Global Imbalances in the Twenty-First Century

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## Abstract

We use a sufficient statistic approach to quantify the general equilibrium effects of population aging on wealth accumulation, expected asset returns, and global imbalances. Combining population forecasts with household survey data from 25 countries, we measure the *compositional effect* of aging: how a changing age distribution affects wealth-to-GDP, holding the age profiles of assets and labor income fixed. In a baseline overlapping generations model this statistic, in conjunction with cross-sectional information and two standard macro parameters, pins down general equilibrium outcomes. Since the compositional effect is positive, large, and heterogeneous across countries, our model predicts that population aging will increase wealth-to-GDP ratios, lower asset returns, and widen global imbalances through the twenty-first century. These conclusions extend to a richer model in which bequests, individual savings, and the tax-and-transfer system all respond to demographic change.

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# 1 Introduction

The world is experiencing rapid demographic change. The average share of the population above 50 years of age has increased from 15% to 25% since the 1950s, and it is expected to rise further to 40% by the end of the twenty-first century (Figure 1, Panel A). There is a widespread view that this aging process has been an important driver of three key macroeconomic trends to date. According to this view, an aging population saves more, helping to explain why wealth-to-GDP ratios have risen and average rates of return have fallen (Figure 1, Panels B and C).<sup>1</sup> Insofar as this mechanism is heterogeneous across countries, it can further explain the rise of global imbalances (Figure 1, Panel D).

Beyond this qualitative consensus lies substantial disagreement about magnitudes. For instance, structural estimates of the effect of demographics on interest rates over the 1970–2015 period range from a moderate decline of less than 100 basis points (Gagnon, Johannsen and López-Salido 2021) to a large decline of over 300 basis points (Eggertson, Mehrotra and Robbins 2019).<sup>2</sup> Turning to predictions for the future, economists are starkly divided about the direction of the effect. Some structural models predict falling interest rates going forward (e.g. Gagnon et al. 2021, Papetti 2019). At the same time, an influential hypothesis argues, based on the dissaving of the elderly, that aging will eventually push savings rates down and interest rates back up. This argument, popular in the 1990s as the “asset market meltdown” hypothesis (Poterba 2001, Abel 2001), was recently revived under the name “great demographic reversal” (Goodhart and Pradhan 2020). In the words of ECB chief economist Philip Lane (Lane 2020):

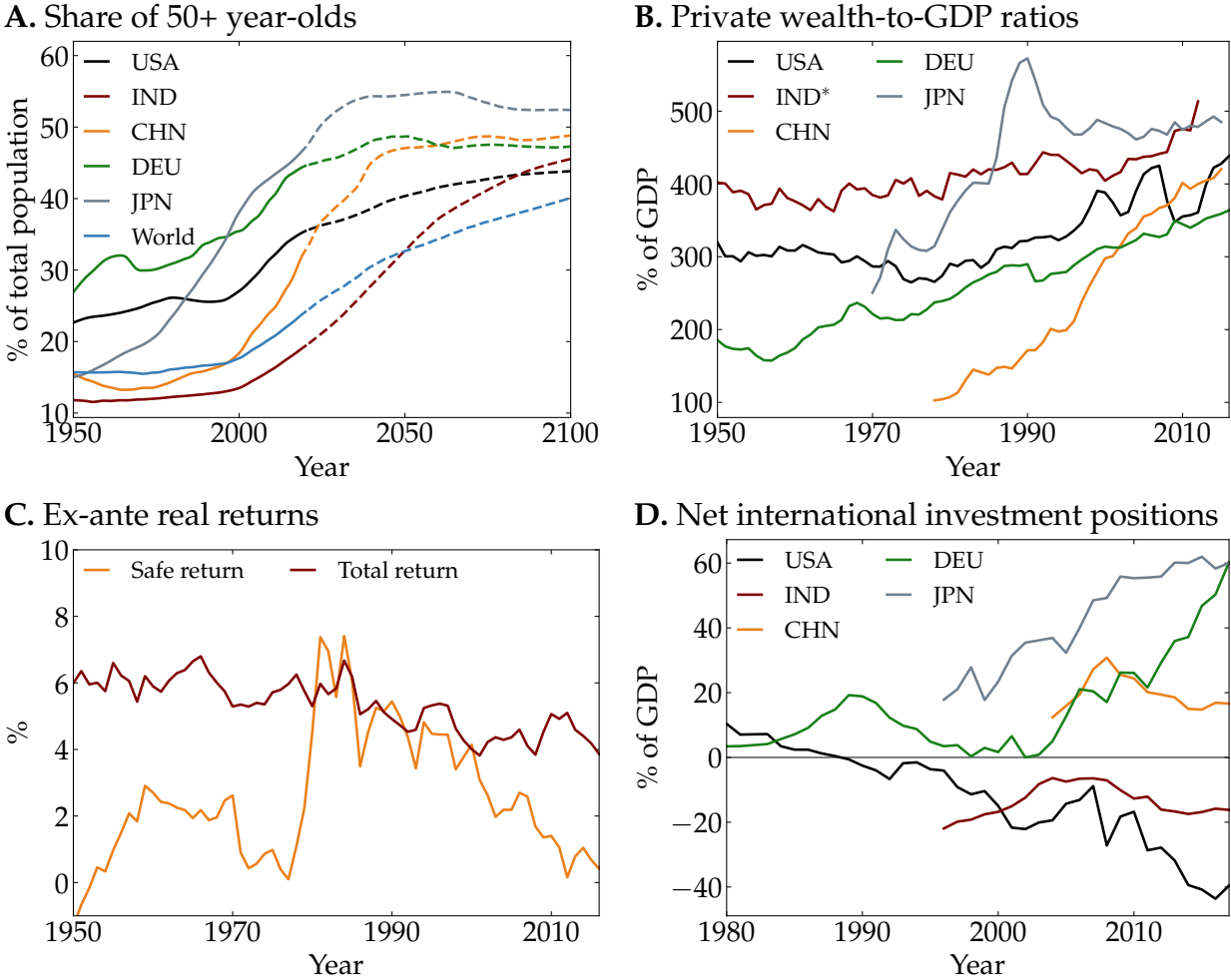
The current phase of population ageing is contributing to the trend decline in the underlying equilibrium real interest rate [...] While a large population cohort that is saving for retirement puts upward pressure on the total savings rate, a large elderly cohort may push down aggregate savings by running down accumulated wealth.

Our paper refutes this argument and shows that, instead, demographics will continue to push strongly in the same direction, leading to falling rates of return and rising wealth-to-GDP ratios. The key to our results is the *compositional effect* of an aging population: the direct impact of the changing age distribution on wealth-to-GDP, holding the age profiles

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<sup>1</sup>We focus primarily on the expected return on total wealth, which we proxy historically by calculating the average return on total wealth, excluding changes in asset valuations. We will often refer to this measure as the “interest rate”; it has been declining since the 1950s. As is well known, safe rates of return have also fallen, though their fall is most pronounced since the 1980s. Appendix A provides more details.

<sup>2</sup>Appendix F presents a selective summary of findings in the literature and shows how to interpret them through the lens of this paper’s framework.



**Figure 1: Demographics, wealth, interest rates and global imbalances**

*Notes:* Panel A presents the share of 50+ year-olds in the five largest economies by GDP and the world as a whole (source: 2019 UN World Population Prospects, the projection is the central scenario). Panel B presents private wealth-to-GDP ratios (source: World Inequality Database; \*: national wealth-to-GDP ratio in India). Panel C presents a measure of the US total return on wealth (orange line) and of the US safe rate of return (red line). Details on the construction of these series are in appendix A. Panel D presents net international investment positions normalized by GDP (source: IMF).

of assets and labor income fixed. In a baseline overlapping generations (OLG) model, this is a sufficient statistic for the actual change in wealth-to-GDP for a small open economy. Further, for a world economy, the compositional effect—when aggregated across countries, and combined with elasticities of asset supply and demand that we obtain with other sufficient statistic formulas—fully pins down the general equilibrium effect on wealth-to-GDP, asset returns, and global imbalances.

We measure the compositional effect by combining population forecasts with household survey data from 25 countries over the period 2016–2100. We find that it is positive

and large everywhere, but also heterogeneous, ranging from an increase in wealth-to-GDP of 48pp in Hungary to 327pp in India. Since the average effect is positive and large, our model shows that there will be no great demographic reversal: through the twenty-first century, population aging will continue to push down global rates of return, with our central estimate being -123bp, and push up global wealth-to-GDP, with our central estimate being a 10% increase, or 47pp in levels. Since the effect is heterogeneous across countries, our model predicts that demographics will also generate large global imbalances. For instance, we find that India's net foreign asset position will steadily grow until it reaches 100% of GDP in 2100, while the United States's net foreign asset position will decline to absorb this demand for assets.

Our sufficient statistic framework offers a transparent way to compute the effect of a changing age distribution on key macroeconomic variables. General equilibrium outcomes can be obtained with a limited amount of information: in addition to the data needed for the compositional effect, we only need data on macroeconomic aggregates and assumptions on two standard macro parameters, the elasticity of intertemporal substitution and the elasticity of substitution between capital and labor. Our framework also clarifies a key limitation of the great demographic reversal hypothesis, which focuses on the decline in one flow (savings) when another (investment) is also declining due to demographic change. In contrast, the compositional effect on stocks (rising wealth-to-GDP) unambiguously implies a falling rate of return.

Our baseline model allows for a broad range of savings motives, but rules out some mechanisms through which population aging can affect behavior. To evaluate how much these can matter, we numerically simulate a richer model in which bequests, individual savings, and the tax-and-transfer system all respond to demographic change. We find that the results are always the same qualitatively, and that with one exception—extreme fiscal adjustments that fall entirely either on tax increases or benefit cuts—they are also close quantitatively to those we obtain directly from our sufficient statistic methodology.

Existing literature has followed two broad approaches, which our paper combines, to quantify the impact of demographic change on macroeconomic outcomes. The first is reduced-form. One branch of this literature, following [Mankiw and Weil \(1989\)](#) and [Poterba \(2001\)](#), computes the effect of a changing age distribution over fixed asset profiles.<sup>3</sup> Another branch, following [Cutler, Poterba, Sheiner, Summers and Akerlof \(1990\)](#) and the “demographic dividend” literature ([Bloom, Canning and Sevilla 2003](#)), computes

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<sup>3</sup>There is also a tradition that computes the effect of changing age distributions over fixed age profiles of savings rates ([Summers and Carroll 1987](#), [Auerbach and Kotlikoff 1990](#), [Bosworth, Burtless and Sabelhaus 1991](#)). This calculation is subject to measurement error and may not give the correct sign of the effect on rates of return, as we show in section 5.

the effect of changing age distributions over fixed income profiles. These “shift-share” calculations are very intuitive, but are not tied to specific general equilibrium counterfactuals. We show that a particular ratio of two such shift-shares is the main determinant of equilibrium outcomes in a fully specified OLG model.

The alternative approach is structural, relying on quantitative general equilibrium OLG models. This tradition, which originated in [Auerbach and Kotlikoff \(1987\)](#), has tackled effects of demographics on aggregate wealth accumulation,<sup>4</sup> asset returns,<sup>5</sup> and international capital flows.<sup>6</sup> Our contribution here is to trace quantitative results back to primitive elasticities, and to the calibration moments that are relevant for the counterfactual of interest. One benefit of this approach is that it can identify the source of conflicting estimates: for instance, the compositional effect in [Gagnon et al. \(2021\)](#) is about the same as in the data, while that in [Eggertsson et al. \(2019\)](#) is about triple that in the data.

In this paper, we focus on the causal effect of projected demographic change in the twenty-first century. We do not explain the underlying sources of this change; instead, we take demographic projections as given. We also rule out some indirect effects of aging, such as changes in total factor productivity or market structure, which are difficult for us to quantify.<sup>7</sup> Although our baseline exercise holds government debt-to-GDP policy fixed, we show how rising government debt can mitigate or even undo the effect of demographic change on real interest rates, while increasing the effect on wealth-to-GDP.<sup>8</sup>

The compositional effects we identify are large, both in the past and in the future. This suggests that demographic change is an important force behind macroeconomic trends. Of course, other developments have also played a major role historically, and our focus on the causal effect of demographic change should not be interpreted as ruling them out.<sup>9</sup>

The paper proceeds as follows. In section 2, we describe our baseline model and define the compositional effect. We show that the effect of aging on wealth-to-GDP in a small

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<sup>4</sup>e.g. [İmrohoroğlu, İmrohoroğlu and Joines \(1995\)](#), [Kotlikoff, Smetters and Walliser \(1999\)](#), [De Nardi, İmrohoroğlu and Sargent \(2001\)](#), and [Kitao \(2014\)](#).

<sup>5</sup>e.g. [Abel \(2003\)](#), [Geanakoplos, Magill and Quinzii \(2004\)](#), [Carvalho, Ferrero and Nechio \(2016\)](#), [Eggertsson et al. \(2019\)](#), [Lisack, Sajedi and Thwaites \(2017\)](#), [Jones \(2020\)](#), [Papetti \(2019\)](#), [Rachel and Summers \(2019\)](#), [Kopecky and Taylor \(2020\)](#), and [Gagnon et al. \(2021\)](#).

<sup>6</sup>e.g. [Henriksen \(2002\)](#), [Börsch-Supan, Ludwig and Winter \(2006\)](#), [Domeij and Flodén \(2006\)](#), [Krueger and Ludwig \(2007\)](#), [Backus, Cooley and Henriksen \(2014\)](#), and [Bárány, Coeurdacier and Guibaud \(2019\)](#).

<sup>7</sup>For the effects of population aging on TFP, see the debate between [Maestas, Mullen and Powell \(2016\)](#) and [Acemoglu and Restrepo \(2017\)](#). For models in which demographics can affect markups via either the structure of consumer demand or firm entry incentives, see [Bornstein \(2020\)](#) vs. [Peters and Walsh \(2019\)](#).

<sup>8</sup>Our quantitative model also shows that increasing the retirement age increases interest rates and reduces wealth-to-GDP, though the magnitude is likely to be modest in practice.

<sup>9</sup>These forces include falling TFP growth, rising inequality, changing risk or liquidity premia, and rising markups. See, for instance, [Rachel and Smith \(2015\)](#), [Eggertsson et al. \(2019\)](#), [Auclert and Rognlie \(2018\)](#), [Straub \(2019\)](#), [Farhi and Gourio \(2018\)](#), and [Eggertsson, Robbins and Wold \(2018\)](#).

open economy exactly coincides with the compositional effect, and that world equilibrium outcomes can be obtained by combining this effect with elasticities of asset supply and demand for which we also derive sufficient statistic formulas. In section 3, we turn to measurement, documenting large and heterogeneous compositional effects across 25 countries for 2016–2100, and calculating their general equilibrium implications. In section 4, we extend the baseline model to capture additional macroeconomic effects of population aging and show that the results from section 3 are a close fit in nearly all cases. Finally, in section 5 we explain why the great demographic reversal hypothesis’s focus on savings rates is incomplete: although demographic forces will indeed push down net savings rates, this will be overwhelmed by an even larger decline in net investment, leading to a decrease in equilibrium rates of return.

## 2 The compositional effect of demographics

In this section, we set up a benchmark life-cycle model with overlapping generations to study the effects of demographic change. We derive two main theoretical results. First, in a small open economy, demographic change only affects macroeconomic aggregates by changing the age composition of the population. Given a demographic projection, these *compositional effects* can be calculated using data from a single cross-section. Second, in an integrated world economy, the long-run effects of demographic change on wealth accumulation, interest rates, and global imbalances can be obtained by simply combining these compositional effects with macroeconomic aggregates, other cross-sectional statistics, and assumptions about two primitive elasticities.

### 2.1 Environment

Our environment is a world economy with overlapping generations (OLG) of heterogeneous individuals. Time is discrete and runs from  $t = 0$  to  $\infty$ , agents have perfect foresight, and capital markets are integrated. Apart from the global return on assets, all variables and parameters are allowed to vary across countries. Country indices are dropped unless there is a risk of ambiguity.

**Individuals.** At each time  $t$ , a country has a population  $N_t = \sum_j N_{jt}$  growing at rate  $1 + n_t \equiv N_t/N_{t-1}$ , with  $N_{jt}$  being the number of individuals of age  $j$ . Each individual faces an exogenous probability  $f_j$  of surviving from age  $j$  to age  $j + 1$ , so the probability of surviving from birth to age  $j$  is  $F_j \equiv \prod_{k=0}^{j-1} f_k$ . The maximal lifespan is  $J$ , so that  $f_J = 0$ .

For now, we assume that this survival profile is constant over time, and that there is no migration. Hence, the age distribution,  $\rho_{jt} \equiv \frac{N_{jt}}{N_t}$ , only varies over time due to changes in fertility and convergence dynamics.<sup>10</sup>

Individuals supply labor exogenously, face idiosyncratic income risk, and can partially self-insure and smooth income over their life cycle by saving in an annuity. Their effective labor supply is  $\bar{l}(z_j)$ , where  $z_j$  is a stochastic process. Unless stated otherwise, all individual variables at age  $j$  are a function of the whole history of the idiosyncratic shocks  $z_j$ , which we denote  $z^j$ .

Individuals with birth year  $k$  choose sequences of consumption  $c_{jt}$  and annuities  $a_{j+1,t+1}$  for all ages  $j = 0, \dots, J$  (with  $t = j + k$ ) to solve the utility maximization problem

$$\begin{aligned} \max_{\{c_{jt}, a_{j+1,t+1}\}} \mathbb{E}_k & \sum_{j=0}^J b_j F_j \frac{c_{jt}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \\ \text{s.t. } & c_{jt} + f_j a_{j+1,t+1} \leq w_t (1-t) \bar{l}(z_j) + tr(z^j) + (1+r_t) a_{jt} \\ & a_{j+1,t+1} \geq -\bar{a} Z_t, \end{aligned} \quad (1)$$

where  $\bar{a}$  is a borrowing constraint,  $w_t$  is the real wage per efficiency unit of labor at time  $t$ ,  $r_t$  is the return on wealth,  $t$  is the labor tax rate, and  $tr(z^j)$  denotes transfers from the government, including wage-indexed social insurance and retirement transfers, for agents of age  $j$  with a history  $z^j$ . The utility weight at age  $j$  is  $b_j F_j$ , combining the survival probability  $F_j$  and an arbitrary age-specific utility shifter  $b_j$ . Deviations from exponential discounting ( $b_j = b^j$  for some  $b$ ) stand in for age-dependent factors that affect the marginal utility of consumption, such as health status or the presence of children. Hence, this model can capture many of the factors that the literature considers essential to understand savings: agents save for life-cycle reasons, for self-insurance reasons, to cover future health costs, and to provide for their children.<sup>11</sup>

The total wealth held by individuals of age  $j$  is the product of  $N_{jt}$  and the average wealth at age  $j$ ,  $a_{jt} \equiv \mathbb{E} a_{jt}$ . Aggregate (private) wealth  $W_t$  is the sum across age groups:

<sup>10</sup>Convergence dynamics for demographics are sometimes called ‘‘momentum’’. Appendix B.1 shows that that fertility and momentum together account for the majority of population aging during the US demographic transition between 1950 and 2100. Changing mortality and migration contribute to a more limited extent, though their importance rises during the latter part of the transition.

<sup>11</sup>We assume that children live with one of their parents, whose consumption  $c_j$  at age  $j$  includes that of the children they care for. Formally, we set  $b_j = \bar{l}(z_j) = tr(z_j) = 0$  when  $j \leq J^w$ , for a  $J^w$  that denotes the start of working life independent from parents. Given this assumption, children do not consume or accumulate assets until age  $J^w$ .

$$W_t \equiv \mathring{\mathbf{a}} \sum_{j=0}^J N_{jt} a_{jt}. \quad (2)$$

**Production.** There is a single good used for private consumption, government consumption, and investment. The final output  $Y_t$  of this good is produced competitively from physical capital  $K_t$  and effective labor input  $L_t$  according to an aggregate production function  $F$

$$Y_t = F(K_t, Z_t L_t),$$

where  $Z_t \equiv Z_0(1 + g)^t$  captures labor-augmenting technological progress. We assume that  $F$  has constant returns to scale and diminishing returns to each factor. Effective labor input  $L_t$  is a standard linear aggregator

$$L_t = \mathring{\mathbf{a}} \sum_{j=0}^J N_{jt} E^j, \quad (3)$$

where  $E^j$  denotes average effective labor input per person of age  $j$ , capturing variation in experience and hours of work over the life cycle. Capital has a law of motion  $K_{t+1} = (1 - d)K_t + I_t$  where  $I_t$  is aggregate investment, and factor prices equal marginal products. The net rental rate of capital is  $r_t = F_K(K_t/(Z_t L_t), 1) - d$ , and the wage per efficiency unit of labor is  $w_t = Z_t F_L(K_t/(Z_t L_t), 1)$ .

We write  $g_t \equiv Y_t/Y_{t-1} - 1$  for the growth rate of the economy. With a constant  $r_t$  and a stationary population,  $g_t = (1 + g)(1 + n) - 1$ . Otherwise,  $g_t$  also reflects changes in capital intensity and the composition of the population.

**Government.** The government purchases  $G_t$  goods, maintains a constant tax rate on labor income  $t$ , gives individuals state-contingent transfers  $tr(z^j)$  indexed to current wages  $w_t$ , and finances itself using a risk-free bond with real interest rate  $r_t$ . It faces the flow budget constraint

$$G_t + w_t \mathring{\mathbf{a}} \sum_{j=0}^J N_{jt} E^j tr_j + (1 + r_t)B_t = t w_t \mathring{\mathbf{a}} \sum_{j=0}^J N_{jt} E^j + B_{t+1}, \quad (4)$$

where a positive  $B_t$  denotes government borrowing. When demographic change disturbs the balance of aggregate tax receipts and expenditures, the government adjusts  $G_t$  to ensure that the debt-to-output ratio  $\frac{B_t}{Y_t}$  follows a given, exogenous, time path.



**Equilibrium.** Given demographics, government policy, an initial distribution of assets, and initial levels of bonds and capital across countries such that  $F_K - d$  is equal to  $r_0$  in each country, an *equilibrium* is a sequence of returns  $\{r_t\}$  and country-level allocations such that, in each country, individuals optimize, firms optimize, and asset demand from individuals equals asset supply from firms and governments,

$$\dot{\mathbf{a}}_c W_t^c = \dot{\mathbf{a}}_c (K_t^c + B_t^c).$$

Dividing by world GDP  $Y_t$ , the above expression can be written as

$$\dot{\mathbf{a}}_c \frac{Y_t^c}{Y_t} \frac{W_t^c}{Y_t^c} = \dot{\mathbf{a}}_c \frac{Y_t^c}{Y_t} \left( \frac{K_t^c}{Y_t^c} + \frac{B_t^c}{Y_t^c} \right). \quad (5)$$

Defining a country's net foreign asset position as the excess of wealth over capital and bonds,  $NFA_t^c \equiv W_t^c - (K_t^c + B_t^c)$ , (5) states that the average NFA-to-GDP ratio is zero, when countries are weighted by their by GDP.

## 2.2 A small economy aging alone

We first study a small open economy undergoing demographic change, while all other countries have constant demographic parameters. In this case, the economy faces a global rate of return  $r$  which is exogenous and fixed—exogenous because the economy is small, and fixed since all other countries have fixed demography. This can be seen as the limit case when the economy has an arbitrarily small world GDP weight  $\frac{Y_t^c}{Y_t}$ , so that its demand and supply of assets do not affect the world equilibrium condition (5).<sup>12</sup> By studying this case, we can analyze how demographics affect macroeconomic aggregates *directly*, independent of any effects operating through equilibrium adjustments in returns  $r_t$ .

We focus on wealth, and our key finding is that demographic change does not affect the wealth levels within age groups, only the distribution of the population across age groups. Formally, the economy exhibits what we call *balanced growth by age*, where the full distribution of wealth within every age group grows at a constant rate.

**Lemma 1.** *For any fixed  $r$ , a small open economy eventually reaches a balanced growth path by age on which, for each age  $j$ , the full distribution of wealth holdings grows at the same rate  $g$  as*

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<sup>12</sup>To obtain a fixed interest rate, we assume that all other countries  $c' \neq c$  are in demographic steady state given a set of mortality profiles  $f_j^c$  and a common growth rate of newborns  $n$ , where the constant growth rate ensures that countries preserve their relative size over time.

technology. In particular, average wealth at age  $j$  satisfies

$$\frac{a_{jt}}{Z_t} = a_j(r). \quad (6)$$

for sufficiently large  $t$  and some function  $a_j(r)$ . If initial asset holdings reflect optimal choices given the fixed  $r$  (in which case  $a_{j0}/Z_0 = a_j(r)$ ), the economy starts on this balanced growth path, and equation (6) holds for all  $t$  and  $j$ .

*Proof.* See appendix B.2. □

The lemma follows since demographic change does not affect the parameters of individuals' life-cycle problems, once these problems are normalized by productivity. Hence, individuals born at different times make the same normalized asset choices given their age, state, and asset holdings. As the influence of initial asset holdings recedes, we reach a balanced growth path by age. Further, if initial assets are consistent with optimization given  $r$ , we start on this balanced growth path. In that case, which we assume from now on, we have  $a_{jt} = (1 + g)^t a_{j0}$  for all  $t$ .

Given lemma 1, aggregate wealth per person satisfies

$$\frac{W_t}{N_t} = \sum_j \hat{\alpha}_j \rho_{jt} a_{jt} = (1 + g)^t \sum_j \hat{\alpha}_j \rho_{jt} a_{j0} \quad (7)$$

Wealth per person changes with the age composition  $\rho_{jt}$  of the population, and otherwise grows at the technological growth rate  $1 + g$ .

We next derive output per person. A constant global  $r$  implies a constant ratio of capital to effective labor  $k(r)$ , defined by  $F_K(k(r), 1) = r + d$ . Aggregate output is then  $Y_t = Z_t L_t F(k(r), 1)$ , where, from (3), aggregate effective labor is  $L_t = N_t \sum_j \hat{\alpha}_j \rho_{jt} E_j$ . Hence

$$\begin{aligned} \frac{Y_t}{N_t} &= Z_t F(k(r), 1) \sum_j \hat{\alpha}_j \rho_{jt} E_j \\ &= \frac{F(k(r), 1)}{F_L(k(r), 1)} (1 + g)^t \sum_j \hat{\alpha}_j \rho_{jt} h_{j0} \end{aligned} \quad (8)$$

where  $h_{j0} = Z_0 F_L E_j = w_0 E_j$  is equal to average labor earnings of individuals of age  $j$ , and we have used the fact that the initial wage is  $w_0 = Z_0 F_L(k(r), 1)$ .

Taking the ratio of (7) and (8), we find that  $W_t/Y_t$  is proportional to the ratio of  $\sum_j \hat{\alpha}_j \rho_{jt} a_{j0}$  and  $\sum_j \hat{\alpha}_j \rho_{jt} h_{j0}$ . The following proposition summarizes this result.

**Proposition 1.** *On the balanced growth path by age, the wealth-to-GDP ratio satisfies*

$$\frac{W_t}{Y_t} \propto \frac{\int \rho_{jt} a_{j0}}{\int \rho_{jt} h_{j0}}, \quad (9)$$

where  $h_{j0} \equiv \mathbb{E} w_{0j}$  is average pre-tax labor income by age, and  $a_{j0} \equiv \mathbb{E} a_{jt}$  is average asset holdings by age.

The proposition implies that all changes in  $W_t/Y_t$  reflect the changing age composition  $\rho_{jt}$  of the population, given fixed age profiles  $a_{j0}$  and  $h_{j0}$ . Equation (9) implies that the log change in wealth to GDP between year 0 and year  $t$  is given by

$$\log \frac{W_t}{Y_t} - \log \frac{W_0}{Y_0} = \log \frac{\int \rho_{jt} a_{j0}}{\int \rho_{jt} h_{j0}} - \log \frac{\int \rho_{j0} a_{j0}}{\int \rho_{j0} h_{j0}} \equiv D_t^{comp}. \quad (10)$$

The key feature of equation (10) is that  $D_t^{comp}$  can be calculated from demographic projections and cross-sectional data alone, with demographic projections providing  $\rho_{jt}$  and cross-sectional data providing  $a_{j0}$  and  $h_{j0}$ . We call  $D_t^{comp}$  the *compositional effect* of aging on  $W_t/Y_t$ . Proposition 1 shows that, for a small open economy, this equals the log change in  $W_t/Y_t$ . The next section shows that  $D_t^{comp}$  also plays a key role in an integrated world economy.

### 2.3 Many countries aging together

We now study the general case where all countries age together, and  $r_t$  adjusts to clear the global asset market. Using an asset supply and demand framework, we find that demographic change increases global asset demand by exactly the average compositional effect (10). We develop this observation into a sufficient statistic result for long-run outcomes, which can be calculated by combining compositional effects with semielasticities of asset demand and supply. These semielasticities, in turn, can be given closed-form expressions in terms of observables and standard macro parameters.

Our analysis starts from a first order approximation of the world asset market clearing condition (5). To simplify, we assume here that net foreign asset positions are zero at an initial date  $t = 0$ , and that governments target a constant  $B_t^c/Y_t^c$ . We relax these assumptions in appendix B.4. We obtain:

$$\frac{\partial}{\partial c} \frac{Y_0^c}{Y_0} D = \frac{W_t^c}{Y_t^c} = \frac{\partial}{\partial c} \frac{Y_0^c}{Y_0} D \frac{K_t^c}{Y_t^c}, \quad (11)$$

where  $D$  denotes level changes between time 0 and  $t$ . The left of (11) is the change in global asset demand, while the right is the change in global asset supply.

We focus on changes between time 0 and the “long run”  $LR$ , when the world has converged to a demographic steady state. Denote by  $e^{c,d} \equiv \frac{\eta \log(W^c/Y^c)}{\eta r}$  the semielasticity of country  $c$ 's aggregate asset demand to the rate of return,<sup>13</sup> and by  $e^{s,c} \equiv -\frac{\eta \log((K^c+B^c)/Y^c)}{\eta r} = \frac{h}{r_0+d} \frac{K_0^c}{W_0^c}$  its semielasticity of asset supply, where  $h$  denotes the elasticity of substitution between capital and labor. For changes between  $t = 0$  and  $t = LR$ , equation (11) then becomes

$$\bar{D}_{LR}^{comp} + \bar{e}^d \cdot (r_{LR} - r_0) \simeq -\bar{e}^s \cdot (r_{LR} - r_0), \quad (12)$$

where bars denote averages across countries using initial wealth shares  $w^c \equiv W_0^c/W_0$  (see the proof of proposition 2 in appendix B.3 for a derivation).

Equation (12) shows that demographics affect equilibrium outcomes by shifting out the asset demand curve by the average compositional effect. In this sense, the compositional effect summarizes the full demographic “shock” to the world equilibrium. Aggregate outcomes are obtained by filtering this shock through the semielasticities  $\bar{e}^d$  and  $\bar{e}^s$ . Solving (12) for  $r_{LR} - r_0$ , we obtain the following proposition.

**Proposition 2.** *If agents start on a balanced growth path by age, initial net foreign asset positions are zero, and governments maintain debt-to-GDP ratios constant, the long-run change in the rate of return is, up to a first order approximation,*

$$r_{LR} - r_0 \simeq -\frac{1}{\bar{e}^d + \bar{e}^s} \bar{D}_{LR}^{comp}, \quad (13)$$

where  $\bar{e}^s = \frac{h}{r_0+d} \frac{\bar{K}_0}{\bar{W}_0}$  is the average semielasticity of asset supply to  $r$ , and  $\bar{e}^d$  is the average semielasticity of individual asset holdings to  $r$ . The wealth-weighted average log change in the wealth-to-GDP ratio is given by

$$\overline{D_{LR} \log \frac{W}{Y}} \simeq \frac{\bar{e}^s}{\bar{e}^s + \bar{e}^d} \bar{D}_{LR}^{comp} \quad (14)$$

*Proof.* See appendix B.3. □

Intuitively, the average compositional effect  $\bar{D}_{LR}^{comp}$  creates an excess demand for assets at fixed  $r$ , which must be absorbed by an increase in the world capital stock and/or a reduction in asset accumulation. If  $\bar{e}^s + \bar{e}^d$  is large,  $r$  falls little, because capital and assets

<sup>13</sup>Formally,  $e^{d,c}$  is the derivative with respect to  $r$  of the balanced growth level of  $\log W/Y$  in a small open economy with exogenous  $r$ , evaluated at the long-run steady-state age distribution. This includes both the direct individual asset accumulation response to  $r$ , and the indirect response from the effect of  $r$  on wages. We discuss  $e^{d,c}$  further in the next section.

are very sensitive to  $r$ . If  $\frac{e^s}{e^s + e^d}$  is large, wealth rises a lot, because a large share of the adjustment occurs through increases in the capital stock rather than through a reduction in asset accumulation.

Beyond interest rates and wealth levels, our framework also speaks to global imbalances. To see why, note first that absent an adjustment in  $r$ , the net foreign asset position (NFA) of a country would increase one-for-one with its compositional effect. In equilibrium,  $r$  must fall to ensure that NFAs are zero on average, so the adjustment in  $r$  has to reduce the average NFA by the average compositional effect. Hence, the change in a country's NFA is determined by the difference between its compositional effect and the average compositional effect, subject to an additional adjustment when countries have different semielasticities to  $r$ . The following proposition summarizes this result.

**Proposition 3.** *Given the conditions of proposition 2, the long-run change in country  $c$ 's net foreign asset position  $NFA^c$  satisfies*

$$\log \left[ 1 + \frac{D_{LR} NFA^c / Y^c}{W_0^c / Y_0^c} \right] \simeq D_{LR}^{comp,c} - \bar{D}_{LR}^{comp} + e^{d,c} + e^{s,c} - \bar{e}^d + \bar{e}^s \quad (r_{LR} - r_0) \quad (15)$$

*Proof.* See appendix B.3. □

Since we have no direct way to predict the effect of demographics on long-run government debt targets, propositions 2 and 3 both assume a benchmark where each country keeps long-run debt-to-GDP constant. Appendix B.4 discusses alternative settings where debt-to-GDP changes in response to demographics. Two special cases stand out: when each country increases its debt-to-GDP target by the amount of its compositional effect, and when each country increases debt-to-GDP by the *average* world compositional effect. In the first case, there is no change in interest rates or net foreign assets, and each country's wealth increases by exactly its compositional effect. In the second case, the same conclusions hold for interest rates and wealth, but net foreign assets in each country increase by the difference between its compositional effect and the global average, leaving the global imbalances predicted by proposition 3 intact.<sup>14</sup>

## 2.4 The asset demand semielasticity $e^d$

Propositions 2 and 3 show that the compositional effects determine aggregate outcomes given the set of asset supply and demand semielasticities  $e^s$  and  $e^d$ .<sup>15</sup> The asset supply

<sup>14</sup>This second case can be viewed as the limit of a specification where we make long-term debt-to-GDP highly responsive to interest rates, taking  $\eta(B^c/Y^c)/\eta r$  uniformly to  $-\infty$  across all countries.

<sup>15</sup>In this section we drop the country superscripts  $c$  for convenience. Subscripts  $c$  denote consumption.

semielasticity  $e^s$  is only a function of observables and of the elasticity of substitution between labor and capital  $h$ .

The asset demand semielasticity  $e^d$  is more challenging to obtain. As noted by [Saez and Stantcheva \(2018\)](#), there is a “paucity of empirical estimates” for how long-run asset accumulation responds to changes in the rate of return.<sup>16</sup> Remarkably, however, in a version of our model without income risk and borrowing constraints, it is possible to express  $e^d$  only in terms of macroeconomic aggregates, the observed age profiles of assets and consumption, and the elasticity of intertemporal substitution  $s$ . The latter is a standard macro parameter that has been the topic of an extensive empirical literature.

To build intuition, we first study the case where technology is Cobb-Douglas and  $r = g$  in the initial steady state. In that case, our result takes a simple form:

$$e^d = s \frac{C}{\underbrace{\frac{(1+g)W}{Z}}_{\equiv e_{substitution}^d} \frac{1+r}{1+r}} + \frac{EAge_c - EAge_a}{\underbrace{\frac{1+r}{Z}}_{\equiv e_{income}^d}}. \quad (16)$$

Here,  $Age_a$  and  $Age_c$  are random variables that capture how asset holdings and consumption are distributed across different ages. The random variables range over ages  $j$ , with probabilities proportional to assets and consumption at each age.<sup>17</sup> Thus,  $VarAge_c$  is large when consumption is spread out across different ages, and  $EAge_c - EAge_a$  is large if consumption, on average, occurs at higher ages than asset holdings do.

In appendix B.5, we derive equation (16), connecting it to the broader logic of life-cycle problems and the cross-sectional outcomes that they produce. The substitution effect  $se_{substitution}^d$  scales with the elasticity of intertemporal substitution, and is proportional to  $VarAge_c$  since there is more scope for intertemporal substitution if consumption is more spread out over the life cycle. The income effect  $e_{income}^d$  reflects the fact that a higher  $r$  increases total income. The size of the increase is proportional to total wealth  $W$  and accrues at an average age of  $EAge_a$ , and it is used to increase consumption by a uniform proportion across all ages, implying that the rise in consumption occurs at an average age of  $EAge_c$ . Aggregate wealth increases if  $EAge_a$  is lower than  $EAge_c$ , because then, on average, the extra interest income is saved before it is consumed.

<sup>16</sup>An elasticity of this kind is important in a variety of contexts, including capital taxation ([Feldstein 1978](#), [Saez and Stantcheva 2018](#)), the response of interest rates to automation ([Moll, Rachel and Restrepo 2021](#)), and the welfare implications of increasing the public debt ([Aguiar, Amador and Arellano 2021](#)). See section 3.2 for a discussion of empirical estimates.

<sup>17</sup>Formally, we define the probability mass of  $Age_a$  at each age  $j$  to be  $p_j a_j / A$ , the share of assets in the cross-section held by people of age  $j$ , and likewise for  $Age_c$ . For the case  $g = 0$ , this is equivalent to defining the mass as the share of assets held at age  $j$  across the life cycle, but with the cross-sectional definition our result holds more generally.

For the more general case, there are two complications. First, when technology is not Cobb-Douglas, the labor share changes with  $r$ , introducing a new term. Second, our previous result relied on current values being the same as present values normalized by growth, which is no longer true when  $r \neq g$ . Writing  $\hat{r} \equiv \frac{1+r}{1+g} - 1$ , we define the present value versions of aggregates:  $W^{PV} \equiv \hat{a}_j \frac{\rho_j a_j}{(1+\hat{r})^j}$  and  $C^{PV} \equiv \hat{a}_j \frac{\rho_j c_j}{(1+\hat{r})^j}$ , and  $Age_a^{PV}$  and  $Age_c^{PV}$  as random variables having probability masses at  $j$  proportional to  $\frac{\rho_j a_j}{(1+\hat{r})^j}$  and  $\frac{\rho_j c_j}{(1+\hat{r})^j}$  respectively. This leads us to the following proposition.

**Proposition 4.** *Consider a small open economy with a steady-state population distribution  $\rho$ . If individuals face no income risk or borrowing constraints, the long-run semielasticity of the steady-state  $W/Y$  to the rate of return is given by*

$$e^d \equiv \frac{\eta \log W/Y}{\eta r} = s e_{substitution}^d + e_{income}^d + (h-1) e_{laborshare}^d. \quad (17)$$

When  $\hat{r} = 0$ ,  $e_{substitution}^d$  and  $e_{income}^d$  are given by (16). When  $\hat{r} \neq 0$ ,

$$e_{substitution}^d = \frac{1}{1+r} \frac{C}{(1+g)W} \frac{E Age_c - E Age_c^{PV}}{\hat{r}} \quad (18)$$

$$e_{income}^d = \frac{1}{1+g} \frac{\frac{C/C^{PV}}{W/W^{PV}} - 1}{\hat{r}} \quad (19)$$

In both cases,  $e_{laborshare}^d$  is given by

$$e_{laborshare}^d \equiv \frac{(1-s_L)/s_L}{r+d}, \quad s_L \equiv \frac{wL}{Y}. \quad (20)$$

*Proof.* See appendix B.5. □

Proposition 4 provides, to our knowledge, the first expression for the semielasticity of aggregate asset demand in a rich quantitative model as a function of measurable sufficient statistics. Earlier work has instead relied on numerical simulations (e.g. Summers 1981, Evans 1983, Cagetti 2001, Aguiar et al. 2021). While the literature has pointed out that this elasticity can be affected by idiosyncratic income uncertainty, we show in section 4 that our formula still provides a close approximation in that context. Further, the results of proposition 4 are continuous at  $\hat{r} = 0$ , so that for small  $\hat{r}$ , (16) is a good approximation to the actual  $e_{substitution}^d$  and  $e_{income}^d$ .<sup>18</sup>

<sup>18</sup>  $e_{laborshare}^d$  tends to be small enough that for  $h$  close to 1, its contribution is insignificant.

### 3 Measurement and implications

This section uses the framework provided by propositions 1–4 to quantify the impact of demographics on macroeconomic aggregates. First, we combine demographic projections with representative household surveys to measure the compositional effect  $D_t^{comp}$  in 25 countries. Second, we use information on age profiles of consumption and wealth together with assumptions on the structural elasticities  $h$  and  $s$  to calculate the semielasticities of asset supply and demand to interest rates. Finally, we combine these results to forecast interest rates, wealth levels, and global imbalances until the end of the twenty-first century.

#### 3.1 The compositional effect

**Implementation.** We take age distributions  $p_{jt}$  from the historical data and future projections of the United Nations World Population Prospects. For these projections, we consider three different scenarios, corresponding to the UN’s baseline projection as well as their “high fertility” and “low fertility” scenarios.

For the age profiles of labor income and wealth, we use representative household surveys. We use labor income data from the Luxembourg Income Study (LIS), which provides harmonized labor surveys for a wide range of countries; we use wealth data from a collection of wealth surveys such as the US Survey of Consumer Finances (SCF) and the European Household Finance and Consumption Survey (HFCS). Our exercise starts in 2016 and the surveys are from this year whenever possible; otherwise, we use the closest available year. See appendix table A.1 for a complete list of data sources and survey years.

For labor income, the model object  $h_{j0}$  is the 2016 average pretax labor income of individuals of age  $j$ . We calculate it using a comprehensive measure of labor income earned by all individuals of age  $j$ —including wages, salaries, bonuses, fringe benefits, and self-employment income before social security and labor income taxes—expressed as a ratio to the number of individuals of age  $j$ .

For assets, the model object  $a_{j0}$  is the 2016 average individual net worth of individuals of age  $j$ . We measure it as total assets net of liabilities, with housing<sup>19</sup> and defined

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<sup>19</sup>The fact that households accumulate assets in part through housing does not change proposition 1, though it potentially changes the general equilibrium implications in propositions 2 and 3. In a simple model, the demand for housing is proportional to overall consumption rather than labor, meaning that the consumption-to-output ratio would appear in the ratio of asset supply to GDP, and a shift-share for this ratio would appear together with the compositional effect in (12). This could mildly attenuate the effects on real interest rates and NFAs.



contribution pension wealth included as assets, and mortgages included as liabilities. For the United States, we also add age-specific estimates of the funded component of the empirically important private defined benefit (DB) pension plans.<sup>20</sup> We map the household wealth measure from the surveys to an individual measure by splitting wealth equally across the head of household, the spouse, and any other household members who are at least as old as the head.<sup>21</sup>

We use the demographic projections and the age profiles of asset and labor income to project the compositional effect from 1950 to 2100 for the twenty-five countries in our sample. To aid interpretation, we sometimes express  $D_t^{comp}$  in terms of predicted changes in the *level* of wealth-to-GDP (in percentage points), rewriting (10) as

$$\frac{W_t}{Y_t} - \frac{W_0}{Y_0} = \frac{W_0}{Y_0} e^{D_t^{comp}} - 1, \quad (21)$$

with  $t = 0$  corresponding to 2016. In this expression,  $W_0/Y_0$  is defined as the aggregate net private wealth to gross domestic product ratio, obtained from either the World Inequality Database (WID) or the OECD.<sup>22</sup>

**Results.** The results from this calculation are displayed in figure 2. Between 1950 to 2016, the compositional effect is positive in all countries, with an average increase of 80pp of GDP, and an increase of 105pp in the United States. These effects are quantitatively large. As a point of comparison, the actual changes in  $W/Y$  that occurred over this period were 220pp for the average country with available data in the WID, and 118pp for the US.

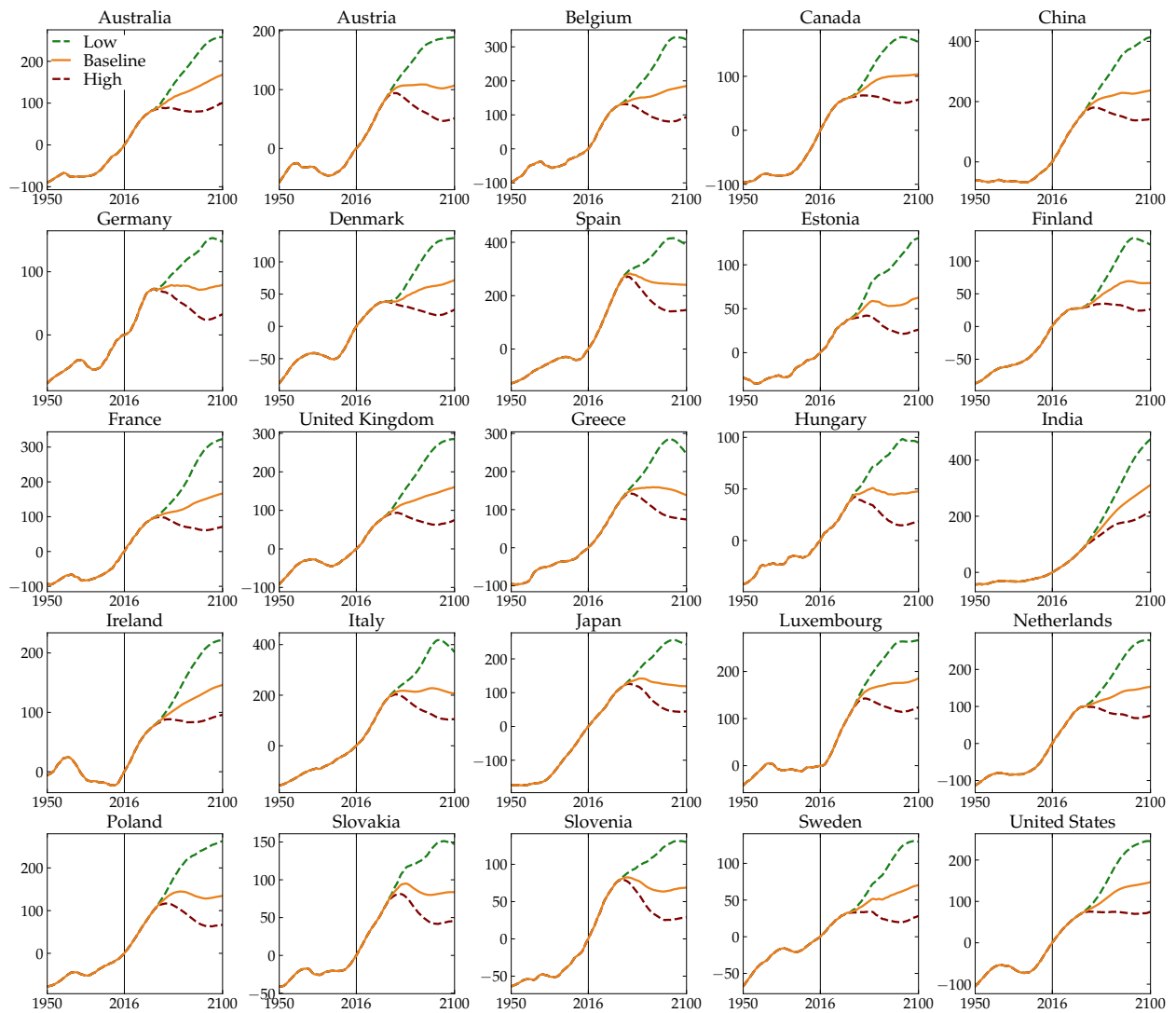
Looking ahead from 2016 to 2100, the effects remain positive, are even larger on average, and are heterogeneous across countries, ranging from 48pp in Hungary to 237pp in China and 327pp in India, with a 147pp increase in the United States. In the high fertility

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<sup>20</sup>For the present value of all DB wealth by age, we use estimates provided by [Sabelhaus and Volz \(2019\)](#), and we set the funded share to 37.5% to ensure consistency with the aggregate amount of non-federal funded defined benefit assets in the US economy. We exclude unfunded DB liabilities since they do not affect the level of wealth  $a_{j0}$  that goes into asset demand; conceptually, we instead think of unfunded DBs as a future transfer  $tr_j$  in the household budget constraint (1). For the same reason, we do not include “social security wealth” in  $a_{j0}$  ([Sabelhaus and Volz 2020](#), [Catherine, Miller and Sarin 2020](#)).

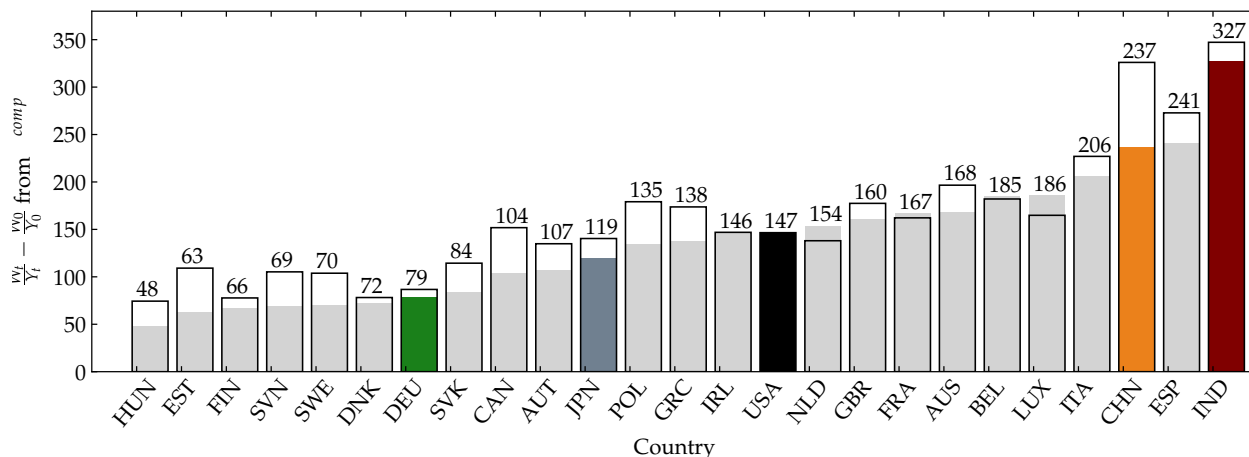
<sup>21</sup>Appendix C.2 shows that the results are robust to using different splitting rules, or to constructing income and wealth at the household level, and combining this with demographic projections for the age distribution of the heads of households.

<sup>22</sup>Net private wealth is defined as the sum of housing, business, and financial assets, net of liabilities, owned by households and nonprofit institutions serving households. Housing assets include the value of dwellings and land; financial assets include currency, bonds, deposits, equity, and investment fund shares, as well as life insurance and private pension funds. In appendix table A.1 we compare private wealth from aggregate data to the aggregated sum of individual survey wealth. In theory, these should be equal, by equation (2). In practice, when the two differ, equation (21) implicitly rescales wealth proportionately at each age so that the survey aggregate matches the WID or OECD total.



**Figure 2:** Predicted change in  $W/Y$  from the compositional effect

*Notes:* This figure depicts the evolution of the predicted change in wealth-to-GDP from the compositional effect, calculated using equation (21) for  $t = 1950$  to 2100, reported in percentage points. The base year is 2016 (vertical line). The solid orange line corresponds to the medium fertility scenario from the UN, the dashed green line to the low fertility scenario, and the dashed red line to the high fertility scenario.



**Figure 3:** Compositional effects and contribution from demographics alone

*Notes:* The solid bars show, for each country, the predicted change in wealth-to-GDP from the compositional effect between 2016 and 2100 in pp, calculated using equation (21), and also reported on top of the bars. These values correspond to the end point of Figure 2. The transparent bars correspond to the case where  $D^{comp}$  in equation (10) is calculated using the US age profiles  $a_{j0}$  and  $h_{j0}$ , but country-specific age distributions  $p_{jt}$ .

scenario, the effect is reduced by a younger population: it is brought down to 75pp in China and to 142pp in the United States; in contrast, the low fertility scenario sees even sharper aging, and the effect swells to 245pp in the United States and 447pp in China.

Figure 3 provides more detail on the heterogeneity across countries, with the solid bars displaying the predicted compositional change in  $W/Y$  to 2100 for the main population scenario. In principle, this cross-country heterogeneity could reflect either differences in demographic evolution or differences in the age profiles of assets and labor income. While both matter, the former is the main factor: countries with large effects are those whose demographic transitions are later and faster. The transparent bars in figure 3 illustrate this by showing similar cross-country heterogeneity in compositional effects if we counterfactually assume that all countries have the same asset and income profile as the United States.<sup>23</sup>

**Unpacking the compositional effect: the case of the United States.** The compositional effect reflects the interaction between population aging and the shapes of the wealth and income profiles. To help explain the magnitudes that we find, we study the case of the United States in greater detail.

The main mechanisms are summarized in figure 4. The grey bars show the evolution

<sup>23</sup>By contrast, appendix figure A.4 shows that countries tend to experience similar compositional effects if they are all assumed to experience US demographics.

A. Changing population distributions over a fixed 2016 age-wealth profile

B. Changing population distributions over a fixed 2016 age-labor income profile

Figure 4: US age-wealth and labor income profiles with population age distributions

Notes: The solid lines in Panel A show the 2016 US age-wealth profiles from the SCF, expressed in current USD. The solid lines in panel B show the 2016 age-income profile from the LIS (CPS), expressed in current USD. Bars represent age distributions: 1950 age distribution in the left panels, 2016 age distribution in the middle panels, and 2100 age distribution in the right panels.

of the population distribution, starting young in 1950 and growing progressively older over time. In the figure, this population evolution is superimposed with the 2016 profiles of assets and labor income, with panel A illustrating how demographic change pushes up assets by moving individuals into high asset ages, and panel B illustrating how demographic change first pushes up aggregate labor income as the baby boomers reach middle age—the so-called “demographic dividend” (Bloom et al., 2003)—and later pushes down aggregate labor income as more individuals reach old age.

The total compositional effect can be separated into contributions from assets and labor supply using a first-order approximation of equation (10):

$$D_t^{\text{comp}} \approx \frac{\Delta p_{jt} - p_{j0} a_{j0}}{\frac{\Delta p_{j0} a_{j0}}{Z^{\text{compa}}}} + \frac{\Delta p_{jt} - p_{j0} h_{j0}}{\frac{\Delta p_{j0} h_{j0}}{Z^{\text{comph}}}} \quad (22)$$

A. Wealth pro le effect

B. Income pro le effect

C. Compositional effect

Figure 5: Effects of demographic composition on  $W$  and  $Y$ : United States 1950-2100

Notes: This figure shows the evolution of the two terms in equation (22). Panel A presents the contribution from the wealth pro le,  $\frac{W_0}{Y_0} D_t^{\text{compa}}$ . Panel B presents the contribution from the labor income pro le,  $\frac{W_0}{Y_0} D_t^{\text{comph}}$ . Panel C presents the overall compositional effect from equation (21), which is approximately equal to the sum of panel A and panel B, overlaid with historical data from the WID. In all graphs, the solid orange line corresponds to the baseline fertility scenario, the dashed green to the low fertility scenario, and the dashed red line to the high fertility scenario of the 2019 UN World Population Prospects. A bootstrapped 95% confidence interval is computed by resampling observations 10,000 times with replacement.

The terms  $D_t^{\text{compa}}$  and  $D_t^{\text{comph}}$  capture the covariances between the changes in age distribution on the one hand, and asset holdings and labor incomes on the other hand.  $D_t^{\text{compa}}$  is positive if the share of people in high asset ages increases, and  $D_t^{\text{comph}}$  is positive if the share of people in high labor income ages decreases. Since old people hold relatively more assets and work relatively less, aging eventually makes both terms positive.

Figure 5 displays the evolution of  $D_t^{\text{compa}}$  and  $D_t^{\text{comph}}$  (multiplied by  $W_0/Y_0$  to obtain level effects on wealth-to-GDP).<sup>24</sup> Panel A shows that  $D_t^{\text{compa}}$  monotonically pushes up the wealth-to-GDP ratio throughout the sample period. The trend attenuates towards the end of the 21st century as aging becomes concentrated in very old ages where asset accumulation ceases. However, the trend never reverses, due to the well-known fact that asset decumulation in old age is quite limited. A large literature has debated the extent to which this limited decumulation reflects life-cycle forces, late-in-life risks, or bequest motives (see e.g. [Abel 2001](#), [Ameriks and Zeldes 2004](#), [De Nardi, French and Jones 2010](#), [De Nardi, French, Jones and McGee 2021](#)); our sufficient statistic result allows us to be agnostic about the exact cause within our given class of possible explanations.<sup>25</sup>

Panel B shows  $D_t^{\text{comph}}$  falling between 1970 and 2010 and then increasing through-

<sup>24</sup>Since  $\frac{W_0}{Y_0} e^{D_t^{\text{comp}}} = 1 + \frac{W_0}{Y_0} D_t^{\text{comp}}$ ,  $\frac{W_0}{Y_0} D_t^{\text{compa}} + \frac{W_0}{Y_0} D_t^{\text{comph}}$ , the two effects approximately sum to the total predicted change from equation (21).

<sup>25</sup>Our benchmark model captures late-in-life risks if  $b_j$  increases in old age. It rules out bequests, but when we allow for them in section 4, we find that the compositional effect remains the primary determinant of the effect on  $W/Y$  at constant interest rates.

out the rest of the 21st century, eventually adding 30pp to the wealth-to-GDP ratio. This non-monotonic pattern is a mirror image of the literature on the so-called “demographic dividend”, which finds a non-monotonic output effect of aging as the population distribution moves across the hump-shaped profile of labor earnings (Bloom, Canning and Sevilla 2003; Cutler et al. 1990). Our findings complement this literature by connecting the output effect of demographics to an inverted effect on the wealth-to-GDP ratio. Quantitatively, this effect contributes a third of the full increase in  $D^{\text{comp}}$  for the United States between 2016 and 2100.

Our results relate to earlier findings by Poterba (2001), who used a shift-share analysis with population projections until 2050 and data from the 1983–1995 waves of the SCF to conclude that  $D^{\text{compa}}$  (which he called “projected asset demand”) would be stable beyond 2020. He used this result to argue that an asset market meltdown was unlikely. In contrast to Poterba, we find a substantial increase in  $D_t^{\text{compa}}$  throughout the remainder of the twenty-first century, reflecting our use of later SCF waves, and, more importantly, population projections with narrower age bins. In addition, Poterba's analysis abstracted from the labor supply term  $D^{\text{comp h}}$ , which we find is not trivial.

For other countries, the logic behind  $D^{\text{comp}}$  is broadly similar to that for the United States. In our online appendix,<sup>26</sup> we reproduce Figures 4 and 5 for all twenty-five countries in our sample. While each country has its own peculiarity—for instance, the timing of the demographic dividend is very uneven—in all of them, aging pushes individuals into higher-asset, lower-income age groups after 2050.

**Robustness to base year and construction of age profiles.** In using a single cross-section of asset and labor income profiles, our calculations rely on our model's property that age profiles are stable over time and grow at a constant rate  $g$ . Given this feature, any cross-section will imply the same compositional effect, and cross-sectional estimates of  $a_{j0}$  and  $h_{j0}$  will agree with estimates of age effects from a time-age-cohort decomposition of repeated cross-sections, provided growth loads on time rather than on cohort effects.

In appendix C.2, we explore the effects of using different base years for the cross-sectional profiles of labor income and asset holdings. For the United States, we use the twelve waves of the LIS going back to 1976, as well as 21 waves of the SCF going back to the 1950s.<sup>27</sup> Calculating  $D_t^{\text{comp}}$  for all the 252 combinations of profiles, we find that the projections for 2016 to 2100 are very stable for all waves of the SCF going back to

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<sup>26</sup>available at [http://web.stanford.edu/~aauclet/demowealth21\\_country\\_appendix.pdf](http://web.stanford.edu/~aauclet/demowealth21_country_appendix.pdf)

<sup>27</sup>For the older waves, we use the SCF+ data developed in Kuhn, Schularick and Steins (2020), which harmonizes and reweights the historical SCF data to maximize the comparability with the modern waves of the SCF. We thank Amir Su for this suggestion.

1989. If we use the pro les from even older waves, we find somewhat smaller effects: for instance, the log compositional effect  $D^{\text{comp}}$  is 20.8% with the oldest pro les, as opposed to 27.8% with the 2016 pro les.<sup>28</sup> In contrast, using the age effects from our time-age-cohort decomposition on the 1989–2016 data leads to an even larger  $D^{\text{comp}}$  of 29.2%.<sup>29</sup>

Hence, while there is some variation across specifications, the effect is always large, positive, and of generally stable magnitude. In the appendix, we show that our results are also robust to using different methods of allocating household wealth to individuals.

### 3.2 Asset supply and demand semielasticities

We now turn to calculating the semielasticities of asset supply and demand using the formulas in proposition 2 and 4.

Asset supply semielasticity  $\bar{e}^s$ . The global asset supply semielasticity captures the response of the capital-output ratio to the required rate of return.<sup>30</sup> Proposition 2 provides a closed-form solution for this semielasticity,

$$\bar{e}^s = \frac{h}{r_0 + d} \frac{\bar{K}_0}{\bar{W}_0}, \quad (23)$$

showing that  $\bar{e}^s$  is proportional to the initial global capital-wealth ratio  $\frac{\bar{K}_0}{\bar{W}_0}$ , the inverse of the user cost of capital  $r_0 + d$ , and the elasticity of substitution between capital and labor  $h$ . From our world economy calibration in section 4, we obtain  $\frac{\bar{K}_0}{\bar{W}_0} = 0.78$  and  $r_0 + d = 9.7\%$ . Given these numbers,  $\bar{e}^s$  is between 4 and 12 for  $h$  in a plausible range from 0.5 to 1.5; it is 8 with a Cobb-Douglas aggregate production function ( $h = 1$ ).

Asset demand semielasticity  $\bar{e}^d$ . The global asset demand semielasticity reflects how much aggregate asset accumulation responds in the long-run to changes in  $r$ . Proposition 4 expresses  $e^{c,d}$  in each country  $c$  as a function of cross-sectional observables, the elasticity of intertemporal substitution (EIS)  $s$ , and capital-labor substitution  $h$ . If  $s$  and  $h$  are common across countries, these semielasticities aggregate as:

$$\bar{e}^d = s \bar{e}_{\text{substitution}}^d + \bar{e}_{\text{income}}^d + (h - 1) \bar{e}_{\text{laborshare}}^d \quad (24)$$

<sup>28</sup>Since  $W/Y = 4.28$  in 2016, a log effect of 27.8% is the same as a level effect of  $4.28^{0.278} - 1 = 137$  p.p.

<sup>29</sup>The choice of pro les matters more for the 1950–2016 compositional effect: there, the oldest pro les give an effect of 12.7%, the newest pro les give an effect of 24.6%, and the time-age-cohort decomposition gives an effect of 35.8%.

<sup>30</sup>This reflects the absence of rents and the constant level of  $B_t/Y_t$ . If there are rents and debt-to-output responds to  $r$ , the responses of capitalized rents and debt to  $r$  also affects  $\bar{e}^s$ .

where bars denote cross-country averages weighted by initial wealth levels.

Implementing the formulas for  $e^{d,c}$  in proposition 4 using the terminal age distribution at 2100, we obtain  $\bar{e}_{\text{substitution}}^d = 39.5$ ,  $\bar{e}_{\text{income}}^d = -2$  and  $\bar{e}_{\text{laborshare}}^d = 5.5$ . Since  $\bar{e}_{\text{substitution}}^d$  is positive and much larger than the other terms,  $\bar{e}^d$  is positive unless the EIS is extremely low.<sup>31</sup> When  $s$  has a reasonable value of 0.5 and the aggregate production function is Cobb-Douglas,  $\bar{e}^d$  is around 18. This means that an exogenous decrease of the interest rate by one percentage point reduces the world wealth-to-GDP ratio by 18%.

The formulas in proposition 4 rely on the distribution of consumption and wealth across different age groups  $j$ , as well as the “present value” equivalent distribution which discounts all values of the age group  $j$  by  $1/(1+\hat{r})^j$ , where  $\hat{r} = \frac{1+r}{1+g} - 1$  is the interest rate net of the economy's growth rate. We construct these distributions by weighting the wealth and consumption profiles by the long-run (2100) population distribution. The wealth profiles by age are obtained as in section 3, and the consumption profiles are backed out from the household budget constraint (1) given the age profiles of wealth and income.<sup>32</sup> The implied distributions are presented in appendix figure A.5.

The distributions can be used together with equation (16) to explain the forces behind our substitution and income effect terms.<sup>33</sup> In (16), the substitution term is approximately a wealth-weighted average of  $C/W$  times the variance of the age of consumption: since  $C/W$  is around 1/6 and consumption is approximately uniformly distributed between ages 20 and 80, a back-of-the-envelope calculation suggests that  $\bar{e}_{\text{substitution}}^d$  is approximately  $(80-20)^2/(12 \cdot 6) = 50$ , close to the actual value of 39.5. The income effect is approximately the difference between the average age of consumption and the average age of assets: since consumption on average occurs a few years before asset holdings, we obtain a negative, but relatively small, income effect.

Finally, our labor share term  $\bar{e}_{\text{laborshare}}^d$  is the cross-country average of  $(1-s_L)/s_L \cdot 1/(r_0+d)$  weighted by wealth. The labor shares from our world economy calibration in section 4 are on average approximately 2/3. Given  $r_0+d = 9.7\%$ ,  $\bar{e}_{\text{laborshare}}^d$  is roughly  $(1-s_L)/s_L \cdot 1/(r_0+d) = (1/2) \cdot 10 = 5$ .

**Comparison to existing empirical estimates.** A number of papers have used variation in capital income taxes to estimate how asset accumulation responds to rates of return

<sup>31</sup>For example, with a Cobb-Douglas production function,  $\bar{e}^d$  is positive when  $s > 2/39.5 = 0.05$ , a relatively weak condition. In comparison, Achdou, Han, Lasry, Lions and Moll (2021) obtain  $s > 1$  as a sufficient condition for  $\bar{e}^d > 0$  in a standard Aiyagari model.

<sup>32</sup>We use this indirect procedure, rather than consumption surveys directly, since the latter tend to be less comprehensive than wealth surveys and are not available for all countries in our study.

<sup>33</sup>Our exact implementation uses equations (18) and (19) and a country-specific  $\hat{r}$ , but in practice  $\hat{r}$  is small enough in every country that expression (16) gives a useful approximation.



(e.g. Kleven and Schultz 2014, Zoutman 2018, Jakobsen, Jakobsen, Kleven and Zucman 2020, Brühlhart, Gruber, Krapf and Schmidheiny 2021). Reviewing this literature, Moll et al. (2021) identify a range for  $e^d$  of 1.25 to 35.<sup>34</sup> This range coincides closely with that implied by equation (24) for plausible values of  $s$  and  $h$ . None of the existing empirical estimates are negative, in line with our findings on substitution effects dominating income effects. In contrast to the inelasticity predicted by representative-agent models, or overlapping generations models with dynastic altruism motives (Barro 1974), all estimates are sufficiently small to imply that interest rates need to fall substantially to accommodate the large compositional effects in the data, as we quantify more precisely in the next section.

### 3.3 General equilibrium implications

We now put together our calculated compositional effects and semielasticities, using proposition 2 and 3 to obtain long-run general equilibrium changes. Here, we define the long run as 2100.

The rate of return and wealth-to-GDP ratios. Proposition 2 shows that long-run changes in the rate of return and average wealth levels are functions of  $\bar{D}_{LR}^{comp}$ ,  $\bar{e}_{LR}^s$ , and  $\bar{e}_{LR}^d$ .

We estimate  $\bar{D}_{LR}^{comp} \approx \Delta_c w^c D_{2100}^{c,comp} = 32\%$  by taking each country's compositional effects until 2100 from section 3.1, averaged using 2016 wealth levels. Equations (23) and (24) in section 3.2 express  $\bar{e}_{LR}^s$  and  $\bar{e}_{LR}^d$  in terms of capital-labor substitutability  $h$  and the elasticity of intertemporal substitution  $s$ . Our central estimate uses canonical values of  $h = 1$  and  $s = 0.5$ . Given the uncertainty surrounding the value of these parameters, however, we also consider a collection of lower and higher values. For the EIS, we consider a low value of  $s = 0.25$  and a high value of  $s = 1$ , spanning the range typically considered in the macroeconomics literature (e.g. Havránek 2015). For capital-labor substitution, we consider a low value  $h = 0.6$  taken from Oberfeld and Raval (2021), and a high value  $h = 1.25$  taken from Karabarbounis and Neiman (2014).

Table 1 presents our results. The left-hand panel shows the changes in the rate of return, calculated using equation (13), while the right-hand panel shows the average change in log wealth-to-GDP in percent, calculated using equation (14).

We find that the equilibrium return  $r$  unambiguously falls in response to demographic change, refuting the “great demographic reversal” hypothesis (Goodhart and Pradhan,

<sup>34</sup>Moll et al. (2021) discuss the literature's implications for a slightly different semielasticity,  $\frac{\partial \log W}{\partial r}$ . Since underlying micro experiments (mostly wealth taxes) are unlikely to change  $\Delta Y$  differentially across treatment and control, however, this should be equivalent to our  $e^d = \frac{\partial \log(W/Y)}{\partial r}$ .

Table 1: Change in world interest rate and wealth-to-GDP

	A. $r_{LR}$ $r_0$			B. $D_{LR} \log \frac{W}{Y}$		
	s			s		
h	0.25	0.50	1.00	0.25	0.50	1.00
0.60	-3.03	-1.56	-0.79	14.6	7.5	3.8
1.00	-2.00	-1.23	-0.70	16.0	9.9	5.6
1.25	-1.65	-1.09	-0.65	16.5	10.9	6.5

Notes: This table presents predictions for the change in the total return on wealth ( $r$ ) and the wealth-weighted log wealth-to-GDP ( $W/Y$ ) between 2016 ( $t = 0$ ) and 2100 ( $t = LR$ ) using our sufficient statistic methodology. Columns vary the assumption on the elasticity of intertemporal substitution ( $s$ ), rows vary the assumption on the elasticity of capital-labor substitution ( $h$ ). Central estimates are in bold.  $r$  is expressed in percentage points, and wealth in percent ( $100 \log$ ).

2020). This result follows because  $\bar{D}_{LR}^{comp}$  and  $\bar{e}^s + \bar{e}^d$  are both positive for any plausible combination of  $s$  and  $h$ . Intuitively, the compositional effect increases net asset demand, and if  $\bar{e}^s + \bar{e}^d > 0$ , then a fall in  $r$  is required to equalize the world's supply and demand of assets.<sup>35</sup> In our central scenario,  $r$  falls 123 basis points by the end of the twenty-first century; the fall is larger when  $s$  or  $h$  are small, since this limits the responsiveness of asset supply and demand to falling returns.<sup>36</sup>

For our central scenario, average wealth-to-GDP increases by 10%, or approximately 47 percentage points in levels as a share of GDP. While substantial, this increase is smaller than the average compositional effect of 32%, since the equilibrium response from the compositional effect is dampened by a factor of  $\bar{e}^d / (\bar{e}^s + \bar{e}^d) \approx 1/3$ , which is the share of adjustment occurring through increases in investment rather than through reductions in asset accumulation. Intuitively, whenever  $\bar{e}^d > 0$ , the general equilibrium response is smaller than the compositional effect, since households accumulate fewer assets as interest rates fall. Wealth responses are larger when investment is elastic relative to accumulation; that is, when  $h$  is large relative to  $s$ .

Our finding of sizable but not radical increases in future wealth-to-GDP ratios lies between the predictions by [Piketty and Zucman \(2014\)](#) and [Krusell and Smith \(2015\)](#): [Piketty and Zucman](#) argue that a steadily lower population growth rate will lead to a

<sup>35</sup>In section 5, we explain why thinking of equilibrium in terms of flows rather than stocks can lead one to miss this conclusion. Our framework can also be extended to rationalize the effect of demographics in models in which agents hold different kinds of assets that command different returns (see e.g. [Kopecky and Taylor 2020](#)). If, for instance, the compositional effect of aging pushes up the net demand for safe assets, then the equilibrium safe return will tend to fall relative to the equilibrium risky return.

<sup>36</sup>Using numerical simulations, [Papetti \(2019\)](#) presents similar comparative statics. Appendix F.2 shows that the functional form implied by our sufficient statistic formulas fits his results very well.

Figure 6: Long-run NFAs under alternative assumptions for  $s$  and  $h$

Notes: This figure presents predictions for NFAs using our sufficient statistic methodology. The solid bars report  $D_{LR}^{NFA^c}/Y^c$  calculated by applying equation (15), assuming  $s = 0.5$  and  $h = 1$ . The confidence intervals correspond to the maximum and the minimum value obtained from this formula across all possible combinations of  $s$  and  $h$  considered in Table 1. The dots correspond to the demeaned compositional effect,  $D_{LR}^{comp}$   $\bar{D}_{LR}^{comp}$ , the first term in equation (15), which is independent of  $s$  and  $h$ .

surge in  $W/Y$  in the twenty-first century,<sup>37</sup> while Krusell and Smith argue that the predictions from representative agent models of no change in  $W/Y$  are more consistent with empirical responses of savings rates to changes in the growth rate.

Global imbalances. Next, we turn to the evolution of net foreign asset positions. Figure 6 shows the changes between 2016 and 2100 predicted by the formula in proposition 3. The bars display the main results, which feature a large divergence in NFA positions, with India and China experiencing increases of 44 to 125 percentage points and Germany experiencing a decrease of 55 percentage points.

The large divergence of NFAs mainly reflects the large heterogeneity in compositional effects found in section 3.1. By proposition 3, this heterogeneity affects global imbalances through the demeaned compositional effects  $D_{LR}^{comp}$   $\bar{D}_{LR}^{comp}$ , whose direct implications for NFAs (assuming no heterogeneity in  $e^s$  and  $e^d$ ) are plotted as circles in figure 6. The demeaned compositional effects broadly mirror the predicted changes in NFAs.

Compared to the results on  $r$  and wealth, the results on global imbalances are less sensitive to the value of the elasticities  $h$  and  $s$ . As proposition 3 shows, semielasticities only affect global imbalances insofar as they differ across countries. Since changing  $h$  and  $s$  primarily moves semielasticities in parallel across countries, they have a relatively

<sup>37</sup>According to Piketty and Zucman,  $W/Y = s/g$  fits the historical data quite well with a stable savings rate  $s$ . If  $g$  falls from 1.5% to 1%, consistent with a 0.5% forecasted decline in population growth to 2100, then their model predicts a log increase in  $W/Y$  of  $\log(1.5) = 40\%$

1

Figure 7: Using the demeaned compositional effect to project NFAs

Notes: Panel A projects NFA-to-GDP ratios between 2016 and 2100 from demeaned compositional effects, using equation (25). Solid lines are historical developments from Figure 1. Panel B shows the projection from the demeaned compositional effect (x-axis) against the actual change in NFA between 1970 and 2015 (y-axis) for the 18 countries for which the data is available. (Source: Lane and Milesi-Ferretti 2017). The dotted line is a 45° line, the solid line is the ordinary least squares regression line.

limited effect on the differences across countries. In the figure, the confidence bands show the minimum and the maximum prediction as  $\theta$  and  $s$  parameters are varied in the range considered above. With a few exceptions, these bands are quite tight.

The importance of demeaned compositional effects suggests a dynamic projection for NFAs that simply uses the demeaned compositional effect at each point in time: <sup>38</sup>

$$D \frac{NFA_t^c}{Y_t^c} = \frac{W_0^c}{Y_0^c} e^{(D_t^{comp} - \bar{D}_t^{comp})} \quad (25)$$

Panel A of Figure 7 implements this calculation. The solid lines show global imbalances until today for the five large economies discussed in the introduction, and the dashed lines show the projections from equation (25). In the next few decades, we expect to see a widening of existing global imbalances: China's net foreign assets will rise substantially, while those of the US will decline. Although these trends attenuate mid-century, the second half of the 21st century features a conspicuous rise in India's net foreign assets, offset partly by a decline in Germany and Japan, whose demographic transitions at that point are nearly complete. These results trace back to the heterogeneity in compositional effects that we documented in section 3.1, which showed China and India with very large  $D_t^{comp}$

<sup>38</sup>Appendix Figure A.6 instead applies equation (15) at each point, taking into account the interest rate adjustment and the heterogeneity in elasticities across countries.

relative to the world average.

To conclude this section, we explore how well equation (25) captures historical variations in NFAs. The results are shown in panel B in figure 7, with the horizontal axis showing the change in NFAs between 1970 and 2015 predicted by (25), and the vertical axis showing the actual changes. For such a simple exercise, the two lines line up quite well: for instance, Japan had the highest projected rise in its NFA, of around 100pp of GDP, which is actually what occurred over this period. The regression line of actual on predicted NFA changes is close to the 45 degree line. Of course, non-demographic forces are also at play in explaining NFA developments over this period of time—including valuation effects from fluctuations in nominal exchange rates and relative stock market performance, as well as flows into Ireland due to its growing status as a tax haven.<sup>39</sup> But this exercise suggests that demographic change, as captured by compositional effects, is in fact an important driver of global imbalances looking backward. This echoes earlier findings from the structural demographics literature (e.g. Backus et al. 2014 and Bárányi et al. 2019).

## 4 The compositional effect in a quantitative model

In our sufficient statistic analysis so far, we predicted equilibrium outcomes from a small set of parameters and data moments. The underlying model in section 2 was rich in some respects, but it also abstracted from a number of forces that the quantitative demographics literature has found to be important to explain savings: bequest motives, changing mortality, and changes in government taxes, transfers, and retirement policy.

In this section, we extend the baseline model to incorporate these features. We simulate this model numerically, and study how well the sufficient statistic analysis holds up. We find that it remains an excellent guide, both qualitatively and quantitatively. The main exception is when the fiscal adjustment in response to an aging population is one-sided: if the budget is balanced entirely with higher taxes, the aggregate effects of aging become uniformly smaller, while if it is balanced entirely with lower benefits, the effects become uniformly larger.

### 4.1 Extending the model of section 2

The basic setup is the same as in section 2. Below we outline the main new features, and provide details in appendix D.1. We continue to omit the country superscript  $c$  unless

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<sup>39</sup>On the importance of valuation effects for NFAs, see Gourinchas and Rey (2007) and Atkeson, Heathcote and Perri (2021). On the importance of tax havens, see Zucman (2013) and Coppola, Maggiori, Neiman and Schreger (2021)

there is a risk of ambiguity.

For the production sector, we now assume that  $F$  is a CES production function with elasticity  $h$ . We make two modifications to the specification of demographics: survival rates  $f_{jt}$  can vary over time, and there is an exogenous number of migrants  $M_{jt}$  of age  $j$  at time  $t$ .

To allow for a longer working life, we introduce a time-varying retirement policy  $r_{jt}$ . We also introduce bequests governed by non-homothetic preferences, which help explain asset inequality and the limited decumulation of assets at old ages. We remove annuity markets given their limited share in aggregate wealth; individuals self-insure against mortality risks, with assets remaining at death given as bequests. Last, we assume that there is intergenerational transmission of ability. These are all standard features of quantitative OLG models (e.g. [De Nardi 2004](#)).

The new individual problem is

$$\max E_k \sum_{j=0}^J \beta^j F_{jt} \left( \frac{c_{jt}}{1 + \frac{1}{s}} \right)^{\frac{1}{s}} + U Z_t^n \left( 1 - f_{jt} \right)^{\frac{1}{s}} \left( \frac{a_{jt}}{1 + n} \right)^{\frac{1}{s}} \quad (26)$$

$$\text{s.t. } c_{jt} + a_{j+1,t+1} = w_t (1 - t_t) \tilde{y}_{jt}(z_j) (1 - r_{jt}) + tr_{jt}(z_j) + (1 + r_t)[a_{j,t} + b_{jt}^f(z_j)] \quad (27)$$

$$a_{j+1,t+1} \leq \bar{a} Z_t.$$

Compared to the setup in section 2, the second term in the utility function captures preferences for bequests. Bequest preferences have curvature  $\frac{1}{s}$  to allow for non-homotheticity, and are scaled with a constant  $U$ , mortality risk  $1 - f_{jt}$ , and a term  $Z_t^n \frac{1}{s}$  that makes this non-homotheticity consistent with balanced growth. In the budget constraint,  $b_{jt}^f(z_j)$  denotes bequests received. The factor  $r_{jt} \in [0, 1]$  denotes retirement policy, and specifies how much labor individuals of age  $j$  are allowed to supply at time  $t$ .

The individual state  $z_j$  consists of a permanent component  $q$ , which is Markov across generations, and a transient component  $\#_j$ , which is Markov across years, both normalized to have mean 1. Total labor supply is the product of these two components and a deterministic age profile:  $\tilde{y}_{jt}(z_j) = q \#_j$ . Bequests received  $b_{jt}^f(z_j)$  are obtained from pooling all bequests from parents of each type  $q$ , distributing them across ages  $j$  in proportion to a fixed factor  $F_j$ , and across types  $q^0$  in proportion to the intergenerational transition matrix of types  $P(q^0|q)$ .

For government policy, we assume that transfers reflect the social security system and are given by  $tr_{jt}(z_j) = r_{jt} q d_t$ , where  $d_t$  denotes the time-varying replacement rate. The government policy consists of a sequence of retirement policies  $\{r_{jt}\}$  and a scal rule

that targets an eventually converging sequence of government debt  $f \frac{B_t}{Y_t} g$ , where the debt sequence is obtained by dynamically adjusting replacement rates  $d_t$ , taxes  $t_t$  and consumption  $G_t$ .

## 4.2 Asset demand and supply in the extended model

Unlike in the baseline model of section 2, demographic change in our extended model affects individual asset accumulation and labor supply decisions even for a fixed  $r$  by generating variation over time in received bequests  $\tilde{b}_{jt}^r(q)$ , survival rates  $f_{jt}$ , tax and benefit policy  $f t_t, d_t g$ , and retirement policy  $r_{jt}$ . These changes create non-compositional effects on the wealth-to-GDP ratio, and imply that propositions 2 and 3 no longer hold, since these propositions relied on the compositional effect summarizing all effects of demographics.

However, the asset demand and supply framework underpinning these propositions still applies to the extended model, provided that we replace the compositional effect  $D_t^{\text{comp}}^{\text{pc}}$  with the more general notion of a small-open-economy effect  $D_t^{\text{soec}}$ . This effect is defined as the change in the wealth-to-GDP ratio for a small open economy facing a fixed  $r$  over time, with  $D^{\text{soe}} = D^{\text{comp}} \neq 0$  indicating that non-compositional effects are present. We can then prove the following.

Proposition 5. If the wealth holdings of agents start in a steady-state distribution given by  $p_0^c$ , then proposition 2 and 3 hold in the extended model, with  $D^{\text{comp}}^{\text{pc}}$  replaced by  $D^{\text{soec}}$ , where  $D_t^{\text{soec}}$  is defined as the change in the wealth-to-GDP ratio between  $t$  and  $t+1$  in a small open economy equilibrium with a constant rate of return  $r$ .

Proof. See appendix D.2. □

Proposition 5 provides a general framework for interpreting the effects of demographics. In appendix F.1, we use this framework to analyze the findings in Eggertsson et al. (2019) (EMR) and Gagnon et al. (2021) (GJLS), two recent papers that find very different effects of demographics on the real interest rate from 1970 to 2015. EMR's much larger effect is explained primarily by a compositional effect that is much larger in the model than in the data, driven both by a steep age-wealth profile and by an overestimated change in the age composition of the population.<sup>40</sup>

The next two sections calibrate our model and interpret the results through the lens of proposition 5.

<sup>40</sup>EMR's lower semielasticities, especially a low  $e^s$ , also play some role.

### 4.3 Calibration

We calibrate a world economy consisting of the 25 economies from section 3. To obtain parameters for each country, we calibrate a steady-state version of our model to 2016 data. Starting from this steady state, we then simulate the model from 2016 onward given demographic projections.

Steady-state calibration procedure Appendix D.3 spells out the steady-state version of our model, which for the most part is standard.<sup>41</sup> The main calibration parameters and results are displayed in table 2. For parameters that are common across countries, the “All” column displays the world value. Country-specific parameters have a  $c$ -superscript, and the US values are displayed for illustration. Below we summarize the main elements of the calibration, with some supplemental information in appendix D.4.

The real rate of return  $r$  is the 2016 value from figure 1 in the introduction, with the calculation described in appendix A. For the wealth-to-GDP ratio  $W^c/Y^c$ , we use the same data as in section 3. We use data from the IMF to obtain country-specific debt levels  $B^c/Y^c$  and net foreign asset positions  $NFA^c/Y^c$ , adjusted to ensure that  $\sum_c NFA^c = 0$ . The capital-output ratio is obtained residually as  $K^c/Y^c = W^c/Y^c - B^c/Y^c - NFA^c/Y^c$ .<sup>42</sup>

On the production side, we set the elasticity of substitution between labor and capital to unity,  $h = 1$ . Countries have a common labor-augmenting growth rate  $g$  calibrated to the average growth in output per labor unit  $\frac{Y^c}{L^c}$  between 2000 and 2016. The common depreciation rate is calibrated to match aggregate capital consumption from the Penn World Table given the capital stocks calibrated above. Given these parameters, we obtain the investment to output ratio and the labor share in each country from  $\frac{K^c}{Y^c}$  and the country-specific growth rate  $g^c = (1 + n^c)(1 + g) - 1$ .

For government policy, we assume that all countries have a discrete retirement policy, with  $r_j^c = 0$  for  $j < J^c$  and  $r_j^c = 1$  for  $j \geq J^c$ , where  $J^c$  is the retirement age. The retirement age is calibrated to the effective age of labor market exit, which we define using information from the OECD and the labor income profiles.<sup>43</sup> We define the income tax

<sup>41</sup>The main non-standard element is a counterfactual flow of migrants, which we introduce to ensure that the steady state implied by the 2016 birth and death rates can exactly match the observed age distribution in 2016. This method is similar to the one used in Penn Wharton Budget Model (2019), and is one way to address a generic problem in the calibration of steady-state demographic models, which is that observed mortality and population shares are generally inconsistent with a stationary population distribution. This adjustment is only needed in the steady state: to simulate the dynamics after 2016, we use the migration flows given in demographic projections.

<sup>42</sup>Note that the implied  $K/Y$  for the US is high relative to standard measures of capital stock. Our methodology implicitly assumes that unmeasured capital accounts for this gap. An alternative procedure would be to explain the gap using markups.

<sup>43</sup>Our main source is the OECD's data on “effective age of labor market exit” from the OECD Pensions at



Table 2: Calibration parameters

Parameter	Description	US	All	Source
<b>Demographics</b>				
$J^w, J$	Initial and terminal ages		20, 95	
$n^c$	Population growth rate	0.6%		UN World Population Prospects
$p_j^c$	Population distribution			UN
$f_j^c$	Survival probabilities			UN
<b>Returns and assets</b>				
$r$	Real return on wealth		3.9%	Described in appendix A
$W^c / Y^c$	Total wealth over GDP	438%		WID
$B^c / Y^c$	Debt over GDP	106.8%		IMF
$NFA^c / Y^c$	Net foreign assets	35.8%		IMF
$K^c / Y^c$	Capital over GDP	367%		$\frac{W^c}{Y^c}$ $\frac{B^c}{Y^c}$ $\frac{NFA^c}{Y^c}$
<b>Production side</b>				
$I^c / Y^c$	Investment over GDP	30.9%		$\frac{K^c}{Y^c}(d + g^c)$
$a^c$	Constant in prod. fn.	0.356		$(r + d) \frac{K^c}{Y^c} \frac{1}{h}$
$s^{L,c}$	Labor share	0.64		$1 - (r + d) \frac{K^c}{Y^c}$
$d$	Depreciation rate		5.79%	$\dot{a}_c d^c K^c$ (PWT) divided by $\dot{a}_c K^c$
$g$	Technology growth		2.03%	World average 2000-16 from $\frac{Y_t}{\dot{a}_c N_{jt} h_{j0}}$
$h$	K/ L elasticity of subst.		1	Standard
<b>Government policy</b>				
$J^{r,c}$	Retirement age	66		OECD
$G^c / Y^c$	Consumption over GDP	12.5%		Government budget
$\bar{d}^c$	Social security benefits	71.3%		Benefits-to-GDP from OECD
$t^c$	Labor tax rate	31.6%		Balanced total budget
<b>Income process</b>				
$c_e$	Idiosyncratic persistence		0.91	Auclert and Rognlie (2018)
$u_e$	Idiosyncratic std. dev.		0.92	Auclert and Rognlie (2018)
$c_q$	Intergenerational persist.		0.677	De Nardi (2004)
$u_q$	Intergenerational std. dev.		0.61	De Nardi (2004)
$\underline{a}$	Borrowing limit	0		
<b>Preferences</b>				
$s$	EIS		0.5	Standard
$\bar{b}^c$	Discount factor process	1.044		See text
$x^c$	Discount factor process	0.00063		See text
$U^c$	Bequests scaling factor	67.95		See text
$n$	Bequest curvature		1.32	See text

rate  $t$  using OECD data on the average tax wedge on personal earnings. Transfers capture the social security system, and satisfy  $tr^c(z_j) = r_j q d^c$ , where we calibrate the social security system replacement rate  $d^c$  by targeting country-specific benefit-to-GDP ratios net of taxes from the OECD Social Expenditure Database. Government consumption  $G^c/Y^c$  is adjusted to ensure a constant debt-to-output ratio.

For the income process, we use average labor income by age to target the deterministic component of labor supply  $\bar{z}_j$  for all ages before retirement,  $j < J^c$ .<sup>44</sup> For the idiosyncratic term  $z$ , the log transient component follows an AR(1) process over the life-cycle, and the log permanent component follows an AR(1) process across generations. The parameters of these processes are taken from [Auclert and Rognlie \(2018\)](#) and [De Nardi \(2004\)](#). We assume that the distribution of bequests received across ages  $F_j$  is common across countries, and we match it to the age distribution of bequests received in the Survey of Consumer Finances.

The remaining parameters are the elasticity of intertemporal substitution  $s$ , the time preference  $\rho$  and  $\beta_j$ , and the weight and curvature on bequests  $(U, n)$ . We assume that parameters  $s$ ,  $U$  and  $n$  are common across countries. To match country-specific age-wealth profiles, we allow the level shifters  $\beta_j$  to vary across countries according to a quadratic formula,  $\log \beta_j^c = \beta_j \log \bar{\beta}^c + x^c (j - 40)^2$ , where  $x^c = 0$  corresponds to exponential discounting. Our calibration first sets  $s$  to 0.5 in line with section 3. To discipline the common  $U$  and  $n$ , we set them jointly with the parameters of US time discount values  $\beta_j^{US}$  to minimize the squared distance to the US profile of wealth by age and the bequest-to-GDP ratio, subject to the constraint of precisely matching the US aggregate wealth to GDP ratio.<sup>45</sup> For all other countries, we set  $\beta^c$  and  $x^c$  to fit the profile of wealth by age, again subject to the constraint of exactly matching the wealth-to-GDP ratio.

Table 3 summarizes calibration outcomes for the 12 largest economies. The successful fit of the long-run compositional effect  $D^{comp,c}$  reflects the goodness of the labor and wealth profiles. In the appendix, we provide additional information about the calibration, including the fit of labor and wealth profiles and the main parameters for all 25 economies.

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a Glance guide. In seven countries, the age provided by the OECD implies that labor market exit happens after the age at which aggregate labor income falls below implied benefit income. In those cases, we define the latter age as the date of labor market exit. See the appendix for details.

<sup>44</sup>For  $j < J^c$ ,  $\bar{z}_j$  is calibrated from age- $j$  labor earnings, scaled up by  $\frac{LFPR_{j,c}}{LFPR_j}$  to compensate for labor force participation at  $j$  being depressed by retirement. Since  $(1 - r_j)\bar{z}_j = 0$  for all  $j < J^c$ , this value does not matter for steady state, but will matter in simulations where the retirement is increased.

<sup>45</sup>The US bequest-to-GDP ratio is from [Alvaredo, Garbinti and Piketty \(2017\)](#), and in the appendix, we also validate the model to the inequality of bequests taken from [Hurd and Smith \(2002\)](#).

Table 3: World economy calibration

Country	$D^{\text{comp}c}$		Components of wealth			Government policy	
	Model	Data	$\frac{W^c}{Y^c}$	$\frac{B^c}{Y^c}$	$\frac{NFA^c}{Y^c}$	$t^c$	$\frac{Berf}{Y^c}$
AUS	30	29	5.09	0.40	-0.46	0.29	0.04
CAN	21	20	4.63	0.92	0.20	0.31	0.04
CHN	47	45	4.20	0.44	0.25	0.30	0.04
DEU	21	20	3.64	0.69	0.58	0.50	0.10
ESP	42	37	5.33	0.99	-0.74	0.39	0.10
FRA	31	30	4.85	0.98	-0.05	0.48	0.13
GBR	27	26	5.35	0.88	0.08	0.31	0.06
IND	65	56	4.16	0.68	-0.08	0.30	0.01
ITA	34	30	5.83	1.31	-0.02	0.48	0.13
JPN	24	22	4.85	2.36	0.66	0.32	0.09
NLD	34	33	3.92	0.62	0.70	0.37	0.05
USA	32	29	4.38	1.07	-0.36	0.32	0.06

Notes: This table presents key initial (2016) steady state statistics for the 12 largest economies by GDP. The first two columns show the value of the compositional effect  $D^{\text{comp}c}$  in both the model and the data, expressed in percent (100 log). The next columns report the wealth-to-GDP ratio  $W/Y$ , government debt-to-GDP ratio  $B/Y$ , and NFA-to-GDP ratio  $NFA/Y$ . The final two columns report the average tax wedge on labor income  $t$  and retirement-benefit-to-GDP ratio  $Berf/Y$ . Data sources are given in the main text.

#### 4.4 Simulations and results

The steady-state calibration pins down the individual parameters, the production parameters, and the initial state of all economies. To study the effect of demographic change, we feed the economy with paths for all demographic variables from the UN World Population Prospects for 2016 to 2100, assuming a smooth transition to a long-run world demographic steady state from that point onwards.<sup>46</sup> We are interested in how wealth levels, rates of return, and net foreign asset positions evolve, and how this evolution relates to our findings from section 3.

Formally, we assume that the world economy has reached a stationary equilibrium in 2300 and we solve for the transition dynamics between 2016 and 2300. Our experiments hold preferences and the aggregate production function constant, but government policy instruments change over time as aging creates fiscal shortfalls that need to be compensated. In our main specification, we assume that the retirement age in all countries increases by one month per year over the first 60 years of the simulation (in line with CBO's projection for the US), and that the government operates a fiscal rule that keeps the debt-to-output ratio constant by relying equally on tax increases, benefit cuts, and

<sup>46</sup>See appendix D.5 for details about our assumptions for the demographic transition after 2100.

Table 4: Baseline and extended model results: 2016–2100

	Dr	$\overline{D \log \frac{W}{Y}}$	$\bar{D}^{\text{comp}}$	$\bar{D}^{\text{soe}}$	$\bar{e}^d$	$\bar{e}^s$
Sufficient statistic analysis	-1.23	9.9	31.8		17.8	8.0
Preferred model specification	-1.23	10.3	34.1	30.3	17.1	8.0
Alternative model specifications						
+ Constant bequests	-1.18	10.0	34.1	27.0	14.9	8.0
+ Constant mortality	-1.23	10.9	34.1	27.1	13.8	8.0
+ Constant taxes and transfers	-1.33	11.9	34.1	30.1	14.5	8.0
+ Constant retirement age	-1.49	13.4	34.1	34.1	14.6	8.0
+ No income risk	-1.47	13.2	33.9	33.9	13.8	8.0
+ Annuities	-1.33	11.5	34.2	34.2	17.2	8.0
Alternative scaling rules						
Only lower expenditures	-1.29	11.0	34.1	32.6	17.9	8.0
Only higher taxes	-0.88	6.7	34.1	19.4	14.6	8.0
Only lower benefits	-1.50	12.9	34.1	39.1	18.4	8.0

Notes: Dr,  $\overline{D \log \frac{W}{Y}}$ ,  $\bar{D}^{\text{comp}}$ , and  $\bar{D}^{\text{soe}}$  denote the changes in the model simulation between 2016 and 2100, with Dr reported in percentage points and the other three reported in percent ( $100 \times \log$ ).

government consumption reductions.

Changes in  $r$  and  $W/Y$ . Table 4 reports the simulation results for Dr and  $\overline{D \log W/Y}$ , together with the corresponding average compositional effect  $\bar{D}^{\text{comp}}$ , the average small open economy effect  $\bar{D}^{\text{soe}}$ , and the average asset demand and supply semielasticities  $\bar{e}^d$  and  $\bar{e}^s$ .<sup>47</sup> We present results from our preferred model specification on the second line, and reproduce results from the sufficient statistic analysis on the first line as a point of comparison.

Overall, the model results are close to the sufficient statistic analysis, with an identical Dr = -1.23pp in both the model and sufficient statistic analysis, and  $\overline{D \log W/Y} = 10.3\%$  in the model compared to 9.9% in the sufficient statistic analysis. The formulas  $\text{Dr} = \frac{\bar{D}^{\text{soe}}}{\bar{e}^d + \bar{e}^s}$  and  $\overline{D \log W/Y} = \frac{\bar{e}^s}{\bar{e}^d + \bar{e}^s} \bar{D}^{\text{soe}}$  from proposition 5 provide an excellent approximation to the full model results, predicting Dr = -1.21pp and  $\overline{D \log W/Y} = 9.7\%$ . Given the success of the first-order approximation formulas, the close match between the model and the sufficient statistic results reflects three facts: a) the model calibration successfully approximates the average compositional effect  $\bar{D}^{\text{comp}}$ , b) the non-compositional effects of

<sup>47</sup>Here,  $\bar{D}^{\text{comp}}$  is calculated as in section 3, and we construct  $\bar{D}^{\text{soe}}$  by simulating the model for each country given a fixed  $r_0$ . For each country, the semielasticities  $e^{d,c}$  and  $e^{s,c}$  are obtained by perturbing  $r$  at a small open economy steady state constructed with 2100 demographics, and calculating the effect on steady-state  $W/Y$  and  $K/Y$ .

aging,  $\bar{D}^{\text{soe}}$   $\bar{D}^{\text{comp}}$ , are relatively small, and c) the model asset demand sensitivity  $\bar{e}^d$  is relatively close to that implied by proposition 4.<sup>48</sup>

For a), the model calibration closely approximates the compositional effect because it is the three inputs to  $\bar{D}^{\text{comp}}$  in each country: we directly match the initial age profile of income, select parameters that approximate the age profile of wealth, and feed in the exact projected change in the age distribution.

To understand b) and c), we sequentially shut off the forces that distinguish the full model from the baseline model underlying the sufficient statistic result. We do this along six rows in table 4. The first four leave the initial calibration intact but shut off dynamic changes: first holding constant bequests received, then perceived mortality, taxes and transfers, and retirement age.<sup>49</sup> The last two involve changes to the steady-state calibration itself, first shutting off income risk, and then replacing bequests at death with annuities. By the final row, we have nearly recovered the baseline model, with the only difference being that our calibration is not flexible enough to perfectly hit  $\bar{D}^{\text{comp}}$ .

The  $\bar{e}^d$  in the full model is similar to that in the baseline model, reflecting two offsetting forces. Relative to the baseline model, the presence of bequests pushes  $\bar{e}^d$  higher, since the savings response to  $r$  can accumulate across generations (e.g. Barro 1974). The absence of annuities pushes  $\bar{e}^d$  lower, since forcing individuals to self-insure against idiosyncratic mortality risk makes savings less sensitive to  $r$ . In table 4, after both forces are shut off by removing bequests and introducing annuities,  $\bar{e}^d$  is on net almost unchanged.

The non-compositional effect of aging  $\bar{D}^{\text{soe}}$   $\bar{D}^{\text{comp}}$  reflects three small, and partially offsetting, demographic forces. First, bequests push up  $\bar{D}^{\text{soe}}$  as the population is aging, since fewer heirs split each bequest; when bequests received are kept constant,  $\bar{D}^{\text{soe}}$  falls from 30.3% to 27.0%. Second, higher taxes and lower social security benefits push down  $\bar{D}^{\text{soe}}$  on net. When these are held constant and the budget is balanced solely with expenditure cuts,  $\bar{D}^{\text{soe}}$  recovers from 27.1% to 30.1%. Last, the delayed retirement age pushes down  $\bar{D}^{\text{soe}}$  by reducing savings and increasing labor supply; when the retirement age is kept constant,  $\bar{D}^{\text{soe}}$  rises from 30.1% to 34.1%, and now agrees exactly with  $\bar{D}^{\text{comp}}$ .<sup>50</sup>

<sup>48</sup>The  $\bar{e}^s$  are identical across all settings since it is only a function of external parameters and moments that are targeted in the calibration.

<sup>49</sup>To make constant bequests received consistent with equilibrium, we assume that government taxes/augments bequests to keep them at the initial level. For mortality, we assume that the mortality rates perceived by individuals ex ante are held constant, while the population still evolves according to objective mortality rates that can change over time. For all these changes to the model, we assume that governments adjust  $G_t$  to maintain a constant debt level.

<sup>50</sup>One potentially surprising feature of table 4 is that perceived mortality has little effect on asset accumulation. This is largely because bequest preferences and social security reduce the role of longevity as a savings motive; in appendix F, we show that Gagnon et al. (2021) find large effects from mortality reductions in a model where these features are absent.

While robust to most features of the model, the close agreement of  $\bar{D}^{\text{soe}}$  and  $\bar{D}^{\text{comp}}$  does not hold when the scal shortfall is closed in a very one-sided way. If the entire shortfall is closed with higher taxes, less after-tax income is available for saving, and  $\bar{D}^{\text{soe}}$  declines to 19.4%. If the entire shortfall is closed with benefit cuts, individuals must save more to fund their own retirement, and  $\bar{D}^{\text{soe}}$  rises to 39.1%. In our main calibration, the shortfall is covered by an even mix of tax increases, benefit cuts, and spending cuts (which are neutral). The absence of a large effect reflects the offsetting effects of tax and benefit adjustments.<sup>51</sup>

Changes to net foreign asset positions. Appendix figure A.10 illustrates the model's predictions for the change in net foreign asset positions. Almost all changes in NFAs over time are explained by differences in  $D^{\text{soe}}$ , which are in turn mainly driven by differences in compositional effects  $D^{\text{comp}}$ . However, compositional effects do not quite explain all the variation in NFAs: although non-compositional effects  $D^{\text{soe}} - D^{\text{comp}}$  are small on average, they vary somewhat across countries.

Transition dynamics. We numerically solve for the transition dynamics in our extended model, and display the resulting paths of world  $r$  and  $W/Y$  in figure 8. To test the how well the long-run sufficient statistic formulas in propositions 2–3 work at different horizons, we apply them at each date  $t$ , combining the time-varying compositional effects  $\bar{D}_t^{\text{comp}}$  with the long-run elasticities  $\bar{e}^d$  and  $\bar{e}^s$ . As we already know from table 4, the two series nearly coincide by 2100. Their dynamics are also quite similar, but the model predicts a somewhat faster decline in  $r$  and rise in  $W/Y$ . Both phenomena reflect the fact that the long-run semielasticities  $\bar{e}^d$  overstate the short-run response of asset accumulation to interest rates. For  $r$ , this implies that interest rates have to fall more in the short run to clear the asset market. For  $W/Y$ , this implies that asset supply is responsible for more of the adjustment, since the supply adjustment is instantaneous in our model.

Overall, this exercise highlights that calculating  $\bar{D}_t^{\text{comp}}$  at different points in time can be useful to predict general equilibrium transition dynamics, although getting the exact timing right requires a structural model.

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<sup>51</sup>The importance of scal adjustment choices for macroeconomic outcomes has been discussed in the pension reform literature (see, for example, Feldstein 1974, Auerbach and Kotlikoff 1987, and Kitao 2014).

A. Change in world  $r$

B. Change in world  $W/Y$

Figure 8: Transition dynamics for rates of return and wealth

Notes: This figure presents the model change in world interest rate and wealth-to-GDP between 2016 and 2100. Solid lines correspond to simulations from our preferred model specification, dashed lines to the sufficient statistic formulas  $\Delta r = \frac{\bar{D}_t^{\text{comp}}}{\bar{e}^d + \bar{e}^s}$  and  $\frac{W_0}{Y_0} \Delta \log W/Y = \frac{W_0}{Y_0} \frac{\bar{e}^s}{\bar{e}^d + \bar{e}^s} \bar{D}_t^{\text{comp}}$ .  $r$  is in pp and  $W/Y$  in percent.

## 5 Demographic change and savings rates

So far, we have analyzed demographics through the lens of stocks: wealth, capital, and net foreign asset positions. An alternative perspective is to focus on flows: savings, investment, and the current account.

The flow perspective has a long tradition in the literature on aging.<sup>52</sup> One key observation in this literature is that the savings rate is hump-shaped in age, so that as the population continues to age, the aggregate savings rate eventually declines. Observers have made various macroeconomic predictions based on this effect: that aging will raise interest rates (Lane 2020), decrease standards of living by impairing capital accumulation (Bloom, Canning and Fink 2010), or exert inflationary pressure as the number of consumers increases relative to the number of producers (Goodhart and Pradhan 2020).

These predictions are not borne out in our analysis. Instead, we find that aging unambiguously lowers the real interest rate, thereby increasing capital intensity and output.<sup>53</sup> A lower real interest rate also implies less inflationary pressure in any standard model in which this pressure is captured by the natural interest rate.<sup>54</sup>

<sup>52</sup>See, e.g., Summers and Carroll (1987), Auerbach and Kotlikoff (1990), Bosworth et al. (1991), Higgins (1998), and Lane (2020).

<sup>53</sup>GDP per person may still decline overall if the workforce composition effect overwhelms capital deepening, but this is a separate channel that does not go through savings, as is clear from equation (8).

<sup>54</sup>That is, in a version of our model with nominal rigidities, if monetary policy does not fully accommodate the natural rate decline by lowering the intercept of its Taylor rule, actual inflation will decline.

### Figure 9: Compositional effects and savings

Notes: Each bar shows the value of the predicted change in the savings-to-GDP ratio from the compositional effect between 2016 and 2100 across countries, reported in pp. Appendix E details the calculation.

To unpack this apparent contradiction, we return to our baseline model of section 2. We first show that this model also predicts a negative effect of aging on savings rates going forward, in line with the literature discussed above. To do this, we note that the aggregate net private savings rate in a small open economy satisfies

$$\frac{S_t}{Y_t} = \mu \frac{\sum_j a_j p_{jt} s_{j0}}{\sum_j a_j p_{jt} h_{j0}}, \quad (28)$$

where  $s_{j0}$  is average net personal savings by age at date 0 (see appendix E for a proof). Equation (28) shows that holding  $r$  constant, changes in the aggregate savings rate are purely determined by compositional forces, just like with wealth-to-GDP. We can therefore also measure this effect using a shift-share calculation.<sup>55</sup> Figure 9 shows the resulting projected savings rates until 2100. These are indeed negative in all countries.<sup>56</sup>

In panel A of figure 10, which represents steady-state equilibrium between savings and investment, we depict this effect as a leftward shift in the private savings curve. At first glance, this might seem to imply an increase in  $r$ , as represented by the hollow circle. But since demographic change lowers the population growth rate and therefore  $g$ , the other curve—representing net investment and public borrowing—also shifts left, and the

<sup>55</sup>While we could in principle perform this calculation using measured savings rates by age, we prefer instead to express (28) using cross-sectional profiles of assets and income alone. This avoids the amplification of measurement error that stems from taking the difference between two large quantities, disposable income and consumption, that are themselves observed with error.

<sup>56</sup>Figure A.11 shows, however, that in many countries, the effect was positive prior to 2016. This gives some support to the common view that aging of baby boomers has pushed up savings in recent decades.



A. World equilibrium: flows

B. World equilibrium: stocks

Figure 10: World asset market equilibrium

Notes: This figure represents asset market equilibrium in flow space (panel A) and in stock space (panel B). The growth rate  $g$  converts Panel B into Panel A. At given  $r$ , demographics increases  $W/Y$  and lowers  $g$ .

overall effect is a decline in  $r$ .

To understand this result, it is useful to compare to panel B, which depicts steady-state equilibrium between asset demand and supply. Here, only the asset demand curve shifts—to the right—and the unambiguous implication is a decline in  $r$ .<sup>57</sup> But the curves in panel A are identical to panel B, just both multiplied by  $g$ .<sup>58</sup> Hence, although both curves in panel A shift left, the net investment curve shifts left by more, producing the same decline in  $r$  as in panel B.<sup>59</sup>

We conclude that the “flow” view of equilibrium in panel A is in principle just as valid as the “stock” view of equilibrium in panel B, but only if we remember the effect of  $g$  on net investment. Ignoring this effect in the context of demographic change, which can significantly push down long-run  $g$ , may give the wrong sign for the change in  $r$ .

<sup>57</sup>For given  $r$ , standard neoclassical theory implies that  $K/Y$  is not affected by demographic change. In our model,  $B/Y$  is also constant. This is subject to debate, but we note that the effect of demographic change on  $B/Y$  could take either sign: if lawmakers hold debt-to-GDP constant,  $B/Y$  will rise, but if they hold net payments  $(r - g)B/Y$  constant, it will fall.

<sup>58</sup>Net savings-to-GDP is  $S_t/Y_t = (W_{t+1} - W_t)/Y_t$ . In steady state, this is  $S/Y = g(W/Y)$ , since  $W_{t+1} = (1 + g)W_t$ . Similarly, net investment-to-GDP is  $g(K/Y)$  and net public borrowing-to-GDP is  $g(B/Y)$ .

<sup>59</sup>Goodhart and Pradhan (2020) acknowledge that investment can also fall in response to demographics, but argue that savings will fall faster than investment. An important part of their argument is that the labor scarcity caused by aging will trigger a rise in labor-saving investment demand. In our model, the capital-labor ratio does increase in equilibrium, but only because the real interest rate falls. See Moll et al. (2021) for a model in which automation leads to rising real interest rates.

## 6 Conclusion

We project out the compositional effect of aging on the wealth-to-GDP ratio of 25 countries until the end of the twenty-first century. This effect is positive, large and heterogeneous across countries. According to our model, this will lead to capital deepening everywhere, falling real interest rates, and rising net foreign asset positions in India and China financed by declining asset positions in the United States.

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# Appendix to "Demographics, Wealth and Global Imbalances in the Twenty-First Century"

## A Appendix to Section 1

The total return on wealth  $r_t$  for the US from 1950–2016 in panel C of Figure 1 is constructed as follows. We take:

Capital  $K_t$  as total private fixed assets at current cost from line 1 of Table 2.1 in the BEA's Fixed Assets Accounts (FA).

Output  $Y_t$  as gross domestic product from line 1 of Table 1.1.5 in the BEA's National Income and Product Accounts (NIPA).

Wealth  $W_t$  as "net private wealth" from the World Inequality Database (WID).

Net foreign assets  $NFA_t$  as the net worth of the "rest of the world" sector from line 147 of Table S.9.a in the Integrated Macroeconomic Accounts (IMA).<sup>60</sup>

Government bonds  $B_t$  as gross federal debt held by the public, from the Economic Report of the President (accessed via FRED at FYGFD PUB).

The safe real interest rate  $r_t^{\text{safe}}$  as the 10-year constant maturity interest rate—from Federal Reserve release H.15 (accessed via FRED at GS10), extended backward from 1953 to 1950 by splicing with the NBER macrohistory database's yield on long-term US bonds (accessed via FRED at M1333BUSM156NNBR)—minus a slow-moving inflation trend, calculated as the trend component of annual HP-filtered inflation in the PCE deflator, with smoothing parameter  $\lambda = 100$ .

Net capital income  $(s_K Y - dK)_t$  as corporate profits plus net interest and miscellaneous payments of the corporate sector (sum of lines 7 and 8 in NIPA Table 1.13), plus imputed net capital income from the noncorporate business sector, under the assumption that the ratio of net capital income to net factor income (line 11 minus line 17) in the noncorporate business sector is the same as the ratio of net capital income to net factor income (line 3 minus line 9) in the corporate sector.<sup>61</sup>

We then calculate our baseline total return on wealth series as

$$r_t = \frac{(s_K Y - dK)_t + r_t^{\text{safe}} B_t}{W_t - NFA_t} \quad (\text{A.1})$$

i.e. as the ratio of net capital income plus real interest income on government debt to domestic assets. This calculation gives the total return on private wealth, excluding changes in asset valua-

<sup>60</sup>This is very similar to the standard net international investment position computed by the BEA, but is chosen because it offers a longer time series.

<sup>61</sup>This imputation is a common way of splitting mixed income within the noncorporate sector between labor and capital, used e.g. by [Piketty and Zucman \(2014\)](#).



A. Baseline

B. With capital gains on fixed assets

C. With imputed returns

D. Return on measured capital

Figure A.1: Alternative ways of constructing the total return on wealth in the US

Notes: Panel A gives our baseline series for the total return on wealth in the US, as described in the text. Panel B adds capital gains on fixed assets, as measured in the fixed assets accounts. Panel C imputes an additional return on unmeasured wealth  $W_t - K_t - B_t - NFA_t$  equal to trend growth. Panel D takes our baseline capital income series and divides it by capital measured in the fixed assets accounts.

tion, under the assumption that the average return on net foreign assets is the same as the average return on private wealth.<sup>62</sup>

This baseline  $r_t$  and its trend are displayed in panel A of [figure A.1](#). The other three panels provide alternative ways to calculate  $r_t$ .

Panel B adds a slow-moving trend of capital good in *ation* minus PCE in *ation*, which we denote by  $\rho_{Kt}$ :

$$r_t = \frac{(s_K Y - dK)_t + r_t^{\text{safe}} B_t + \rho_{Kt} K_t}{W_t - NFA_t}$$

Average in *ation* of goods in the capital stock is inferred by taking the ratio of changes in the nominal stock (FA Table 2.1, line 1) and changes in the quantity index (FA Table 2.2, line 1), and as with PCE in *ation* above, we take the slow-moving trend component using the HP filter with  $\lambda = 100$ . This accounts for expected capital gains on fixed capital (assuming that the expectation follows the trend).

Panel C assumes that there is some unmeasured return on the portion of wealth  $W_t - K_t - B_t - NFA_t$  that cannot be accounted for by capital, bonds, or net foreign assets, which it sets equal to the trend real GDP growth rate  $g_t$ :

$$r_t = \frac{(s_K Y - dK)_t + r_t^{\text{safe}} B_t + g_t (W_t - K_t - B_t - NFA_t)}{W_t - NFA_t}$$

where  $g_t$  is again calculated using the HP filter with  $\lambda = 100$ . If  $W_t - K_t - B_t - NFA_t$  is the capitalized value of pure rents in the economy, for instance, its value might be expected to grow in line with output.

Finally, panel D simply divides net capital income by the measured capital stock:

$$r_t = \frac{(s_K Y - dK)_t}{K_t}$$

Note that despite these alternative constructions, the 1950–2016 trends in panels A, B, and C of [figure A.1](#) are almost identical: -.033, -.033, and -.032 percentage points, respectively. All show a steady decline.

The return on capital in panel D, on the other hand, is quite different: it has a smaller long-term trend decline, of -.022 percentage points per year, and since roughly 1980 it actually displays a mild increase. This post-1980 pattern of a constant or increasing return on capital has been widely remarked upon in the literature—for instance, [Gomme, Ravikumar and Rupert \(2011\)](#), [Farhi and Gourio \(2018\)](#), and [Eggertsson et al. \(2018\)](#). The main source of the disparity between panels A–C and panel D is that the former divides by wealth, while the latter divides only by measured capital. Since our primary object of interest is wealth, we prefer the former convention. Another advantage of using wealth in the denominator is that capital may be imperfectly measured in the fixed assets accounts.

<sup>62</sup>This can be seen by rearranging (A.1) as  $r_t = \frac{s_K Y - dK + r_t^{\text{safe}} B + r NFA}{W}$ , which gives the total return  $r_t$  on private wealth if  $r_t$  equals the return on  $NFA_t$ . We take this route because data on capital income from foreign assets is not comparable to domestic data; for instance, the national accounts only measure dividend payments, not the total net capital income, on foreign equities (other than FDI) held in the US, and also only measure nominal rather than real interest payments on bonds. The trend in  $r_t$ , however, is not very sensitive to alternative assumptions on the average rate for  $NFA_t$ .

## B Appendix to Section 2

### B.1 Contribution of changing fertility to aging, 1950-2100

Figure A.2 uses our model of the age distribution of the population in each country to decompose population aging into contributions from fertility, mortality, migration and the so-called momentum effect. Our measure of population aging is the changes in the share of the population aged 50 or above. Denote by  $Dp$  the change in this share between two periods  $t_0$  and  $t_1$ . To isolate the role of primitive forces for  $Dp$ , we start with an initial age distribution in year  $t_0$ . We obtain the contribution of fertility plus "momentum" by simulating the population distribution holding mortality and migration constant until  $t_1$ , and then computing the counterfactual change  $D^f p$  in the share of the 50+ year-old in this scenario. The ratio  $D^f p / Dp$  gives us the contribution of fertility and momentum to population aging, which our baseline model of section 2 includes, with the remainder accounted for by mortality and migration, which the baseline model abstracts from. We conduct this exercise over two separate time periods  $t_0$ - $t_1$ : 1950-2016 and 2016-2100.

Figure A.2 presents the results, showing  $D^f p / Dp$  over these two time periods for the 25 countries in our sample. The top panel shows that, between 1950-2016, fertility and momentum contributed an average of 63.5% of population aging. The bottom panel shows that, between 2016 and 2100, their contributions are projected to shrink a little to an average of 55.9%, but still constitute the majority of the contribution. Hence, our baseline assumption of fixed mortality and migration is a useful first pass at the data, although decreasing mortality becomes increasingly important to population aging as we look towards the 21st century. Our model of section 4 allows for time variation in mortality and models the savings response to it.

### B.2 Proofs of lemma 1 and proposition 1

The ratio  $K_t / Z_t L_t$  of capital to effective labor is constant over time, pinned down by constant  $r$  and the condition  $r_t + d = F_K(K_t / (Z_t L_t), 1)$ . From the condition  $w_t = Z_t F_L(K_t / (Z_t L_t), 1)$ ,  $w_t$  is then proportional to  $Z_t$  and grows at the constant rate  $g$ . It follows immediately that average pre-tax labor income  $h_{jt} = E w_t \tilde{w}_j = (1 + g)^t w_0 E \tilde{w}_j$  grows at the constant rate  $g$ .

Letting hats denote normalization of time-subscripted variables by  $(1 + g)^t$ , and defining  $\hat{b}_j = (1 + g)^{j(1 - \frac{1}{s})} b_j$ , the household utility maximization problem (1) becomes

$$\begin{aligned} \max_{\hat{c}_t, \hat{a}_{j+1, t+1}} E_k & \prod_{j=0}^J \hat{a}_j \hat{b}_j F_j \frac{\hat{c}_{jt}^{1 - \frac{1}{s}}}{1 - \frac{1}{s}} \\ \text{s.t. } & \hat{c}_t + (1 + g) \sum_j \hat{a}_{j+1, t+1} = w_0 (1 + g)^t (Z_j) + \text{tr}(z^j) + (1 + r) \hat{a}_{jt} \\ & \hat{a}_{j+1, t+1} = Z_0 \bar{a} \end{aligned} \quad (\text{A.2})$$

This problem is no longer time-dependent: given the same asset holdings  $\hat{a}_j$ , state  $z^j$  and age  $j$ , households optimally choose the same  $(\hat{c}_j, \hat{a}_{j+1})$  regardless of  $t$ . Regardless of their date of birth, every cohort born in this environment will have the same distribution of normalized assets  $\hat{a}_j$  at each age  $j$ . Hence, once  $t$  is high enough that all living agents were born in this environment, there exists a balanced-growth distribution of assets at each age that grows at rate  $g$ . Average assets normalized by productivity satisfy  $\bar{a}_t / Z_t = (E \bar{a}_t) / Z_t = (E \hat{a}_j) / Z_0 = \bar{a}_j(r)$  for some function

A. 1952-2016 change in the share of 50+ : percentage due to fertility and momentum

B. 2016-2100 change in the share of 50+ : percentage due to fertility and momentum

Figure A.2: Contribution of fertility and momentum to population aging

Notes: This figure presents the percentage of the change in the share of 50+ that is due to fertility changes and momentum. It is computed as the ratio between the change in this share under the assumptions of constant mortality rates and migration flows, and under the baseline assumptions for 1952-2016 (panel A) and 2016-2100 (panel B).

$a_j(r)$ . If, at date 0, already-living agents start with the joint balanced-growth distribution of assets and states, then this holds immediately.

The ratio of aggregate wealth to aggregate labor at time  $t$  is

$$\frac{W_t}{L_t} = \frac{\hat{a}_j N_{jt} a_{jt}}{\hat{a}_j N_{jt} E_j} = \frac{\hat{a}_j N_{jt} (1+g)^t a_{j0}}{\hat{a}_j N_{jt} h_{j0} / w_0} = (1+g)^t w_0 \frac{\hat{a}_j p_{jt} a_{j0}}{\hat{a}_j p_{jt} h_{j0}} \quad (\text{A.3})$$

The ratio of output to aggregate labor is

$$\frac{Y_t}{L_t} = \frac{F(K_t, Z_t L_t)}{L_t} = Z_t F\left(\frac{K_t}{Z_t L_t}, 1\right) = Z_t F\left(\frac{K_0}{Z_0 L_0}, 1\right) \quad (\text{A.4})$$

where we use the fact that the capital-to-effective-labor ratio is constant. Dividing (A.3) and (A.4), the wealth-to-output ratio is

$$\frac{W_t}{Y_t} = \frac{w_0}{Z_0 F(K_0 / Z_0 L_0, 1)} \frac{\hat{a}_j p_{jt} a_{j0}}{\hat{a}_j p_{jt} h_{j0}} \quad (\text{A.5})$$

where the first factor is constant with time. We conclude that  $\frac{W_t}{Y_t}$  grows in proportion to  $\frac{\hat{a}_j p_{jt} a_{j0}}{\hat{a}_j p_{jt} h_{j0}}$ .

### B.3 Proofs of propositions 2 and 3

**Proof of proposition 2.** Within each country  $c$ , for a constant rate of return  $r$ , lemma 1 shows that there exists a balanced-growth distribution of assets normalized by productivity. Assuming we start with this balanced-growth distribution, then at each  $t$ , (A.5) implies

$$\begin{aligned} \frac{W_t^c}{Y_t^c} &= \frac{w_0^c}{Z_0^c F^c(K_0^c / Z_0^c L_0^c, 1)} \frac{\hat{a}_j p_{jt}^c a_{j0}^c}{\hat{a}_j p_{jt}^c h_{j0}^c} \\ &= \frac{F_L^c(K_0^c / Z_0^c L_0^c, 1)}{F^c(K_0^c / Z_0^c L_0^c, 1)} \frac{\hat{a}_j p_{jt}^c a_{j0}^c}{\hat{a}_j p_{jt}^c h_{j0}^c} \\ &= \frac{F_L^c(k^c(r), 1)}{F^c(k^c(r), 1)} \frac{\hat{a}_j p_{jt}^c a_{j0}^c}{\hat{a}_j p_{jt}^c h_{j0}^c} = \frac{W^c}{Y^c}(r, p_t^c) \end{aligned}$$

where  $p_t^c = p_{jt}^c g_j$ , and  $k(r)$  is the capital-to-effective-labor ratio associated with  $r$ , defined implicitly by  $F_K^c(k(r), 1) = r + d$ .

Each country's share of world GDP is then given by

$$\frac{Y_t^c}{Y_t} = \frac{Z_t^c L_t^c y^c(r)}{\hat{a}_j Z_t^c L_t^c y^c(r)} = \frac{Z_0^c n_t^c y^c(r) \hat{a}_j p_{jt}^c}{\hat{a}_j Z_0^c n_t^c y^c(r) \hat{a}_j p_{jt}^c} \frac{Y^c}{Y}(r, p_t, n_t),$$

where  $n_t^c = N_t^c / N_t$  and  $p_t$  and  $n_t$  denote vectors across all countries, and  $y^c(r) = F^c(k^c(r), 1)$ .

The capital-to-output ratio in every country can also be written as a function of  $r$ ,  $\frac{K^c}{Y^c}(r)$ ,  $k^c(r) / F^c(k^c(r), 1)$ , and we assume that government policy maintains a constant  $\frac{B^c}{Y^c}$  in each country.

We assume that the economy is in balanced growth corresponding to long-run  $r_0$  at date 0, which means that the initial wealth-to-output ratio is  $\frac{W^c}{Y^c}(r_0, p_0^c)$  and that the initial capital-output ratio is  $\frac{K^c}{Y^c}(r_0)$ . We also assume that net foreign asset positions in each country are 0 at time 0, i.e.

that

$$\frac{W^c}{Y^c}(r_0, p_0^c) - \frac{K^c}{Y^c}(r_0) - \frac{B^c}{Y^c} = 0.$$

In the long run,  $p_t^c$  and  $n_t^c$  converge to constants  $p_{LR}^c$  and  $n_{LR}^c$  in each country. Suppose that the real return  $r_t$  converges to a long-run value  $r_{LR}$ . Then the world asset market clearing condition is

$$0 = \dot{a}_c \frac{Y^c}{Y}(r, p, n) - \frac{W^c}{Y^c}(r, p^c) - \frac{K^c}{Y^c}(r) - \frac{B^c}{Y^c} \quad (A.6)$$

which holds for both  $(r, p, n) = (r_0, p_0, n_0)$  and  $(r, p, n) = (r_{LR}, p_{LR}, n_{LR})$ . Subtracting the former from the latter, we have

$$\begin{aligned} 0 &= \dot{a}_c \frac{Y^c}{Y}(r_{LR}, p_{LR}, n_{LR}) - \frac{W^c}{Y^c}(r_{LR}, p_{LR}^c) \\ &\quad - \frac{K^c}{Y^c}(r_{LR}) - \frac{B^c}{Y^c} - \dot{a}_c \frac{Y^c}{Y}(r_0, p_0, n_0) - \frac{W^c}{Y^c}(r_0, p_0^c) - \frac{K^c}{Y^c}(r_0) - \frac{B^c}{Y^c} \\ &= \dot{a}_c \frac{Y^c}{Y}(r_{LR}, p_{LR}, n_{LR}) - \frac{Y^c}{Y}(r_0, p_0, n_0) - \frac{W^c}{Y^c}(r_{LR}, p_{LR}^c) - \frac{K^c}{Y^c}(r_{LR}) - \frac{B^c}{Y^c} \\ &\quad + \dot{a}_c \frac{Y^c}{Y}(r_0, p_0, n_0) - \frac{W^c}{Y^c}(r_{LR}, p_{LR}^c) - \frac{K^c}{Y^c}(r_{LR}) - \frac{B^c}{Y^c} - \frac{W^c}{Y^c}(r_0, p_0^c) - \frac{K^c}{Y^c}(r_0) - \frac{B^c}{Y^c} \end{aligned}$$

Note that  $\frac{W^c}{Y^c}(r_0, p_0^c) - \frac{K^c}{Y^c}(r_0) - \frac{B^c}{Y^c}$  is 0 by the assumption of zero initial NFA. To first-order, therefore, the product of  $\frac{Y^c}{Y}(r_{LR}, p_{LR}, n_{LR}) - \frac{Y^c}{Y}(r_0, p_0, n_0)$  and  $\frac{W^c}{Y^c}(r_{LR}, p_{LR}^c) - \frac{K^c}{Y^c}(r_{LR}) - \frac{B^c}{Y^c}$  is zero as well. To first-order, the above then simplifies to the equivalent

$$\begin{aligned} 0 &= \dot{a}_c \frac{Y_0^c}{Y_0} \frac{W^c}{Y^c}(r_{LR}, p_{LR}^c) - \frac{K^c}{Y^c}(r_{LR}) - \frac{B^c}{Y^c} - \frac{W^c}{Y^c}(r_0, p_0^c) - \frac{K^c}{Y^c}(r_0) - \frac{B^c}{Y^c} \quad (A.7) \\ &= \dot{a}_c \frac{Y_0^c}{Y_0} \frac{W^c}{Y^c}(r_{LR}, p_{LR}^c) - \frac{W^c}{Y^c}(r_0, p_{LR}^c) + \frac{W^c}{Y^c}(r_0, p_{LR}^c) - \frac{W^c}{Y^c}(r_0, p_0^c) - \frac{K^c}{Y^c}(r_{LR}) - \frac{K^c}{Y^c}(r_0) \\ &\quad - \dot{a}_c \frac{Y_0^c}{Y_0} \frac{\frac{W^c}{Y^c}(r_0, p_{LR}^c)}{\frac{W^c}{Y^c}(r_0, p_0^c)} (r_{LR} - r_0) + \frac{W^c}{Y^c}(r_0, p_0^c) \exp(D_{LR}^{comp}) - 1 - \frac{\frac{K^c}{Y^c}(r_0)}{\frac{K^c}{Y^c}(r_0)} (r_{LR} - r_0) \\ &\quad - \dot{a}_c \frac{W_0^c}{Y_0} \frac{\frac{1}{\frac{W^c}{Y^c}(r_0, p_{LR}^c)} \log \frac{W^c}{Y^c}(r_0, p_{LR}^c)}{\frac{1}{\frac{W^c}{Y^c}(r_0, p_0^c)} \log \frac{W^c}{Y^c}(r_0, p_0^c)} (r_{LR} - r_0) + D_{LR}^{comp} \frac{1}{\frac{W^c}{Y^c}(r_0, p_0^c)} \frac{\frac{K^c}{Y^c}(r_0)}{\frac{K^c}{Y^c}(r_0)} (r_{LR} - r_0), \quad (A.8) \end{aligned}$$

where we write  $\frac{Y_0^c}{Y_0}$  and  $\frac{W_0^c}{Y_0}$  to denote  $\frac{Y^c}{Y}(r_0, p_0, n_0)$  and  $\frac{W^c}{Y^c}(r_0, p_0^c, n_0^c)$ . Let us also define

$$\begin{aligned} e^{d,c} &= \frac{\frac{1}{\frac{W^c}{Y^c}(r_0, p_{LR}^c)} \log \frac{W^c}{Y^c}(r_0, p_{LR}^c)}{\frac{1}{\frac{W^c}{Y^c}(r_0, p_0^c)} \log \frac{W^c}{Y^c}(r_0, p_0^c)} \\ e^{s,c} &= \frac{1}{\frac{W^c}{Y^c}(r_0, p_0^c)} \frac{\frac{K^c}{Y^c}(r_0)}{\frac{K^c}{Y^c}(r_0)} \\ w^c &= \frac{W^c}{W}(r_0, p_0, n_0) \end{aligned}$$

and divide both sides of (A.8) by  $\frac{W}{Y}(r_0, p_0, n_0)$  to obtain the first-order result

$$\begin{aligned} 0 &= \frac{W^c}{Y^c} \exp \left[ D_{LR}^{comp,c} + (e^{d,c} + e^{s,c})(r_{LR} - r_0) \right] \\ &= \bar{D}_{LR}^{comp} + (\bar{e}^d + \bar{e}^s)(r_{LR} - r_0) \end{aligned} \quad (\text{A.9})$$

where we let bars denote averages across countries with initial wealth weights  $w^c$ . The equations (12) and (13) are rearrangements of (A.9).

Now, the change in  $W^c/Y^c$  in each country can be written to first-order as

$$D_{LR} \log \frac{W^c}{Y^c} = D_{LR}^{comp,c} + e^{d,c}(r_{LR} - r_0)$$

Summing up both sides with weights  $w^c$ , this becomes

$$\overline{D_{LR} \log \frac{W^c}{Y^c}} = \bar{D}_{LR}^{comp} + \bar{e}^d(r_{LR} - r_0)$$

and using (A.9) to substitute out for  $r_{LR} - r_0$ , we obtain (14),

$$\overline{D_{LR} \log \frac{W^c}{Y^c}} = \frac{\bar{e}^s}{\bar{e}^s + \bar{e}^d} \bar{D}_{LR}^{comp} \quad (\text{A.10})$$

**Proof of proposition 3.** The change in  $NFA^c/Y^c = W^c/Y^c - K^c/Y^c - B^c/Y^c$  is given by

$$\begin{aligned} D_{LR} \frac{NFA^c}{Y^c} &= \frac{W_0^c}{Y_0^c} \exp \left[ D_{LR}^{comp,c} + (e^{d,c} + e^{s,c})(r_{LR} - r_0) \right] - \frac{K_0^c}{Y_0^c} \exp \left[ D_{LR}^{comp,c} + (e^{d,c} + e^{s,c})(r_{LR} - r_0) \right] - \frac{B_0^c}{Y_0^c} \exp \left[ D_{LR}^{comp,c} + (e^{d,c} + e^{s,c})(r_{LR} - r_0) \right] \\ &= \frac{W_0^c}{Y_0^c} \exp \left[ D_{LR}^{comp,c} + (e^{d,c} + e^{s,c}) \frac{\bar{D}_{LR}^{comp}}{\bar{e}^d + \bar{e}^s} \right] - \frac{K_0^c}{Y_0^c} \exp \left[ D_{LR}^{comp,c} + (e^{d,c} + e^{s,c}) \frac{\bar{D}_{LR}^{comp}}{\bar{e}^d + \bar{e}^s} \right] - \frac{B_0^c}{Y_0^c} \exp \left[ D_{LR}^{comp,c} + (e^{d,c} + e^{s,c}) \frac{\bar{D}_{LR}^{comp}}{\bar{e}^d + \bar{e}^s} \right] \\ &= \frac{W_0^c}{Y_0^c} \exp \left[ D_{LR}^{comp,c} + (e^{d,c} + e^{s,c}) (\bar{e}^d + \bar{e}^s) \frac{\bar{D}_{LR}^{comp}}{\bar{e}^d + \bar{e}^s} \right] - \frac{K_0^c}{Y_0^c} \exp \left[ D_{LR}^{comp,c} + (e^{d,c} + e^{s,c}) (\bar{e}^d + \bar{e}^s) \frac{\bar{D}_{LR}^{comp}}{\bar{e}^d + \bar{e}^s} \right] - \frac{B_0^c}{Y_0^c} \exp \left[ D_{LR}^{comp,c} + (e^{d,c} + e^{s,c}) (\bar{e}^d + \bar{e}^s) \frac{\bar{D}_{LR}^{comp}}{\bar{e}^d + \bar{e}^s} \right] \\ &= \frac{W_0^c}{Y_0^c} \exp \left[ D_{LR}^{comp,c} + (e^{d,c} + e^{s,c}) (\bar{e}^d + \bar{e}^s) (r_{LR} - r_0) \right] - \frac{K_0^c}{Y_0^c} \exp \left[ D_{LR}^{comp,c} + (e^{d,c} + e^{s,c}) (\bar{e}^d + \bar{e}^s) (r_{LR} - r_0) \right] - \frac{B_0^c}{Y_0^c} \exp \left[ D_{LR}^{comp,c} + (e^{d,c} + e^{s,c}) (\bar{e}^d + \bar{e}^s) (r_{LR} - r_0) \right] \end{aligned}$$

Rearranged, this gives the desired result, which is

$$\log \left( 1 + D_{LR} \frac{NFA^c}{Y^c} \right) - \log \left( 1 + D_{LR} \frac{K^c}{Y^c} \right) - \log \left( 1 + D_{LR} \frac{B^c}{Y^c} \right) = D_{LR}^{comp,c} + (e^{d,c} + e^{s,c}) (\bar{e}^d + \bar{e}^s) (r_{LR} - r_0)$$

## B.4 Relaxing assumptions in propositions 2 and 3

In the more general case, we allow initial NFA's to be non-zero and debt-to-output ratios to vary over time. Below, we show how the formulas are modified in this case, and some discussions of how particular sequences of debt-to-output ratios can mitigate or even undo the general equilibrium effects on interest rates.

Allowing for nonzero initial NFAs. With non-zero initial NFAs, there is a compositional effect of aging on net asset demand insofar as the change in relative GDP across countries is correlated with initial NFAs.

If  $NFA_0^c$  is not zero in every country  $c$ , we would retain an additional term in (A.7), equal to first-order to

$$\dot{\bar{a}}_c \frac{Y^c}{Y} (r_{LR}, p_{LR}, n_{LR}) - \frac{Y_0^c}{Y_0} \frac{NFA_0^c}{Y_0^c} = \dot{\bar{a}}_c \frac{Y_0^c}{Y_0} D_{LR} \log \frac{Y^c}{Y} (r_{LR}, p_{LR}, n_{LR}) \frac{NFA_0^c}{Y_0^c}$$

When we divide by  $\frac{W_0}{Y_0}$  as in our derivation of (A.9), this becomes

$$\dot{\bar{a}}_c \frac{W^c NFA_0^c}{W_0^c} D_{LR} \log \frac{Y^c}{Y} (r_{LR}, p_{LR}, n_{LR}) \quad (A.11)$$

which will show up as an additional term in (A.9). Since the wealth-weighted average of  $\frac{NFA_0^c}{W_0^c}$  is zero by global market clearing, this can be written as a wealth-weighted covariance

$$\text{Cov}_{wc} \left( \frac{NFA_0^c}{W_0^c}, D_{LR} \log \frac{Y^c}{Y} (r_{LR}, p_{LR}, n_{LR}) \right) \quad (A.12)$$

If we define

$$D_L^{\text{demog}} \frac{Y^c}{Y} = \frac{\eta(\log \frac{Y^c}{Y})}{\eta p} D_{LR} p + \frac{\eta(\log \frac{Y^c}{Y})}{\eta n} D_{LR} n$$

to be the change in GDP shares caused by demographic change alone, holding  $r$  constant, and

$$\bar{e}^{\text{weight}} = \text{Cov}_{wc} \left( \frac{NFA_0^c}{W_0^c}, \frac{\eta(\log \frac{Y^c}{Y})}{\eta r} \right) \quad (A.13)$$

then the modified (A.9) becomes

$$\bar{D}_{LR}^{\text{comp}} + \text{Cov}_{wc} \left( \frac{NFA_0^c}{W_0^c}, D_{LR}^{\text{demog}} \frac{Y^c}{Y} \right) + (\bar{e}^d + \bar{e}^s + \bar{e}^{\text{weight}})(r_{LR} - r_0) = 0 \quad (A.14)$$

and we can solve to obtain

$$r_{LR} - r_0 = \frac{\bar{D}_{LR}^{\text{comp}} + \text{Cov}_{wc} \left( \frac{NFA_0^c}{W_0^c}, D_{LR}^{\text{demog}} \frac{Y^c}{Y} \right)}{\bar{e}^d + \bar{e}^s + \bar{e}^{\text{weight}}}$$

Note that the two departures from our previous result, the covariance in (A.14) and the covariance in the definition (A.13) of  $\bar{e}^{\text{weight}}$ , both involve wealth-weighted covariances between initial net foreign asset positions as shares of wealth,  $\frac{NFA_0^c}{W_0^c}$ , and some change in each country's GDP weight (either in response to demographics or endogenously in response to  $r$ ). A priori, there is no particular reason to have a covariance in either direction here, and indeed we have found that these terms seem quite small in practice, to the point that they can be disregarded in our main analysis without risk for non-trivial errors.

Our previous simplification for the average change in wealth-to-GDP no longer holds, but we can still write

$$D_{LR} \log \frac{W^c}{Y^c} \approx \bar{D}_{LR}^{\text{comp}} + \bar{e}^d (r_{LR} - r_0).$$



The change in NFA in each country is

$$D \log \left( 1 + \frac{D_{LR} NFA^c / Y^c}{W_0^c / Y_0^c} \right) = D_{LR}^{comp} + (e^{d,c} + e^{s,c})(r_{LR} - r_0)$$

Change in debt-to-output ratios. Suppose that each country operates a fiscal rule that targets an exogenous sequence  $\frac{B_t^c}{Y_t^c}$  which converges to some long-run value  $\frac{B_{LR}^c}{Y_{LR}^c}$  in every country. The average change in bonds is a shifter of asset supply, and the new version of (12) is

$$\bar{D}_{LR}^{comp} \frac{\overline{D_{LR} B^c / Y^c}}{W_0^c / Y_0^c} + \bar{e}^d (r_{LR} - r_0) = \bar{e}^s (r_{LR} - r_0), \quad (A.15)$$

where  $\frac{\overline{D_{LR} B^c / Y^c}}{W_0^c / Y_0^c} = \bar{a}_c w^c \frac{B_{LR}^c}{Y_{LR}^c} \frac{B_0^c}{Y_0^c}$  is the average log change in debt-to-output ratios.

We can solve (A.15) to obtain  $r_{LR} - r_0$ , which is simply the original formula with this shifter in supply subtracted from the compositional effect:

$$r_{LR} - r_0 = \frac{\bar{D}_{LR}^{comp} \frac{\overline{D_{LR} B^c / Y^c}}{W_0^c / Y_0^c}}{\bar{e}^d + \bar{e}^s} \quad (A.16)$$

The average change in wealth-to-GDP now becomes

$$D_{LR} \log \frac{W^c}{Y^c} = \frac{\bar{e}^s}{\bar{e}^d + \bar{e}^s} \bar{D}_{LR}^{comp} + \frac{\bar{e}^d}{\bar{e}^d + \bar{e}^s} \frac{\overline{D_{LR} B^c / Y^c}}{W_0^c / Y_0^c} \quad (A.17)$$

which adds the direct impact of increasing debt to (14), and the change in NFA in each country is

$$\log \left( 1 + \frac{D_{LR} NFA_{LR}^c}{W_0^c / Y_0^c} \right) = D_{LR}^{comp} \frac{D_{LR} B^c / Y^c}{W_0^c / Y_0^c} + \bar{D}_{LR}^{comp} \frac{\overline{D_{LR} B^c / Y^c}}{W_0^c / Y_0^c} + (e^{d,c} + e^{s,c}) (\bar{e}^d + \bar{e}^s) (r_{LR} - r_0) \quad (A.18)$$

which now subtracts the change in asset supply from bonds in each country from the compositional effect on asset demand, but is otherwise the same formula as (15).

Neutralizing debt-to-output policy. The equations (A.16) and (A.18) show that effects of demographics on interest rates and NFAs can be neutralized if governments conduct a debt policy that absorbs the shift in aggregate asset demand. More precisely, if all governments expand debt in line with their compositional effect

$$D_{LR}^{comp} = \frac{D_{LR} B^c / Y^c}{W_0^c / Y_0^c}$$

we obtain  $r_{LR} - r_0 = 0$  and  $\log \left( 1 + \frac{D_{LR} NFA_{LR}^c}{W_0^c / Y_0^c} \right) = 0$  for every country  $c$ . Intuitively, if governments in every country expand debt to perfectly meet the new demand for assets, there is no change in net asset demand, so interest rates stay constant and NFAs do not change. In this case, the change in wealth equals the compositional effect in every country, since there is no general equilibrium feedback reducing the impact of increased asset demand on wealth.

An alternative specification is if each government increases the level of its debt-to-output ratio in line with the average compositional effect, so that for all  $c$

$$\frac{W}{Y} \bar{D}_{LR}^{comp} = D_{LR} \frac{B^c}{Y^c}$$

In this case, we still have  $r_{LR} - r_0 = 0$ , but now (A.18) implies that NFAs change in line with the demeaned compositional effect across countries  $D^{comp} - \bar{D}^{comp}$ .

Strikingly, these findings are also true in the transition, not just in the long run. That is, if the sequence of debt holdings satisfies  $\frac{D_t B^c / Y^c}{W_0^c / Y_0^c} = D_t^{comp}$  for every  $t$ , then interest rates and NFAs are constant over time, and the path of wealth-to-output ratios equals the path of the compositional effect. Moreover, if  $\frac{D_t B^c / Y^c}{W_0^c / Y_0^c} = \bar{D}^{comp}$ , then the interest rate change is zero at every point in time, and NFAs at every time period for each country is the demeaned compositional effect.

## B.5 Proof of proposition 4

### B.5.1 Framework

Dropping idiosyncratic risk and the borrowing constraint, and writing assets (which are now common to all individuals of the same age at a given time) as  $a_{j,t}$  for convenience, the individual problem is

$$\begin{aligned} \max_{\{c_{j,t}, a_{j+1,t+1}\}_{j=0}^J} & \sum_{j=0}^J \beta^j F_j \frac{c_{j,t}^{1-\frac{1}{s}}}{1-\frac{1}{s}} \\ \text{s.t. } & c_{j,t} + f_j a_{j+1,t+1} = w_t (1-t)_j + tr_j + (1+r_t) a_{j,t} \end{aligned} \quad (\text{A.19})$$

where  $t = k + j$  is time. Note that we assume agents start and end the lifecycle with zero assets:  $a_{0,t} = 0$  and  $F_{J+1} a_{J+1,t} = 0$ .

The only way in which time-varying macroeconomic aggregates enter this problem is through the real wage  $w_t$  and real interest rate  $r_t$ . Suppose that we have a balanced growth path by age with technology growth  $g$ , so that  $r_t = r$ ,  $w_t = w(1+g)^t$  for some  $w$ , and we can also write  $a_{j,t} = a_j(1+g)^t$  and  $c_{j,t} = c_j(1+g)^t$ . Then (A.19) becomes

$$\begin{aligned} \max_{\{c_j, a_{j+1}\}_{j=0}^J} & \sum_{j=0}^J \tilde{\beta}^j F_j \frac{c_j^{1-\frac{1}{s}}}{1-\frac{1}{s}} \\ \text{s.t. } & c_j + f_j(1+g)a_{j+1} = w y_j + (1+r) a_j \end{aligned} \quad (\text{A.20})$$

where we define  $\tilde{\beta}_j = \beta_j(1+g)^{j(1-\frac{1}{s})}$  and  $y_j = (1-t)_j + tr_j$ . Again, we have the initial and terminal conditions  $a_0 = 0$  and  $a_{J+1} = 0$ .

### B.5.2 Effects on wealth-to-GDP

We are interested in characterizing the semielasticity of steady-state  $W/Y$  with respect to steady-state  $r$ . Using balanced growth by age and a demographic steady state, we have both  $W_t = W(1+g)^t$  and  $Y_t = Y(1+g)^t$ , where  $W = \sum_{j=0}^J p_j a_j$  and  $Y = F(k(r), 1)L_0$ .

Thanks to linearity of the budget constraint and homotheticity of intertemporal preferences,

the entire problem (A.20) scales in  $w$ . Hence, if we use  $W$  to denote aggregate wealth given the normalization  $w = 1$ , then for a different  $w$ , steady-state wealth will be  $wW$ .

We can now write the semielasticity of wealth-to-GDP with respect to  $r$  as

$$\frac{\eta \log(w(r)W(r)/Y(r))}{\eta r} = \frac{\eta \log W(r)}{\eta r} + \frac{\eta \log(w(r)/F(k(r), 1))}{\eta r} \quad (\text{A.21})$$

where the first term  $\frac{\eta \log W(r)}{\eta r}$  is the semielasticity of wealth with respect to  $r$ , holding fixed wages at  $w = 1$ . Note that the second term, the semielasticity of the wage-output ratio with respect to  $r$ , will be zero in the Cobb-Douglas case. We will return to this term for the non-Cobb-Douglas case later, and focus on evaluating the first term  $\frac{\eta \log W(r)}{\eta r}$  for now.

### B.5.3 Budget constraint, Euler equation, and wealth

At the optimum, the budget constraint in (A.20) will hold with equality, and (recalling that we are now using the normalization  $w = 1$ ) can be rewritten as

$$a_{j+1} = \frac{1}{f_j} \frac{1}{1+g} y_j - c_j + (1+r)a_j$$

Multiply both sides by the survival probability  $F_{j+1} = f_j F_j$  to obtain

$$F_{j+1}a_{j+1} = \frac{1}{1+g} F_j y_j - c_j + (1+r)a_j \quad (\text{A.22})$$

Now, a demographic steady-state implies that  $\frac{p_{j+1}}{F_{j+1}} = \frac{1}{1+n} \frac{p_j}{F_j}$ . Multiplying (A.22) by this gives

$$\begin{aligned} p_{j+1}a_{j+1} &= \frac{1}{1+g} p_j y_j - c_j + (1+r)a_j \\ &= \frac{1}{1+g} p_j (y_j - c_j) + p_j (1+\hat{r})a_j \end{aligned} \quad (\text{A.23})$$

where we use the steady-state relationship  $1+g = (1+n)(1+g)$  and the definition  $1+\hat{r} = \frac{1+r}{1+g}$ .

Also using  $F_{j+1} = f_j F_j$ , the optimization problem (A.20) has the Euler equation  $\tilde{b}_j c_j^{1/s} = \tilde{b}_{j+1} \frac{1+r}{1+g} c_{j+1}^{1/s}$ . (Note that survival probabilities drop out, since they appear symmetrically in the price of an annuity and in preferences.) This can be iterated forward to obtain

$$c_j = \frac{\tilde{b}_j}{\tilde{b}_0} \frac{1+r}{1+g} j^{1/s} c_0 \quad (\text{A.24})$$

We can solve for the  $2J+1$  unknowns  $c_0, \dots, c_J$  and  $a_1, \dots, a_J$  (recalling  $a_0 = a_{J+1} = 0$ ) using  $2J+1$  equations, specifically (A.24) for  $j = 1, \dots, J$  and (A.23) for  $j = 0, \dots, J$ .

Note that  $r$  enters these equations in two places: on the right in (A.23) (inside  $1+\hat{r} = \frac{1+r}{1+g}$ ), and on the right in (A.24). To find the derivative of  $\log W$  with respect to  $r$ , we will separately perturb  $r$  in each of these two places, find the effect on  $\log W$ , and then sum to find the overall derivative. The part of the derivative from perturbing  $r$  inside the Euler equation (A.24) can be thought of as the substitution effect, since it takes into account the effect of intertemporal substitution but ignores the effect of  $r$  in the budget constraint, and part from perturbing  $r$  inside (A.23) can be

thought of as the income effect.

We will consider two cases of increasing complexity: first the special case where steady-state  $\hat{r} = 0$  and preferences are Cobb-Douglas, and then the general case, for which we will also need to evaluate the second term in (A.21).

#### B.5.4 Special case with $\hat{r} = 0$ and Cobb-Douglas

Substitution effect. Given steady-state  $\hat{r} = 0$ , we can sum (A.23) from 0 to  $j$  to obtain

$$p_j a_j = \dot{a} \sum_{k=0}^{j-1} p_k \frac{1}{1+g} (y_k - c_k) \quad (\text{A.25})$$

which for  $j = J+1$  becomes the lifetime budget constraint

$$0 = \dot{a} \sum_{j=0}^J p_j \frac{1}{1+g} (y_j - c_j) \quad (\text{A.26})$$

Summing up (A.25), we obtain

$$\begin{aligned} W = \dot{a} \sum_{j=0}^J p_j a_j &= \dot{a} \sum_{j=0}^J \sum_{k=0}^{j-1} p_k \frac{1}{1+g} (y_k - c_k) \\ &= \dot{a} \sum_{j=0}^J (J-j) p_j \frac{1}{1+g} (y_j - c_j) = \dot{a} \sum_{j=0}^J p_j j \frac{1}{1+g} (c_j - y_j) \end{aligned} \quad (\text{A.27})$$

This simple result states that total wealth is the gap between the ages at which consumption occurs and the ages at which (after-tax-and-transfer) income is earned.<sup>63</sup> The intuition is simple: every year that income is deferred for later consumption requires holding an asset.<sup>64</sup>

Now suppose that we perturb  $r$  in (A.24). Log-differentiating gives

$$\frac{dc_j}{c_j} = s_j \frac{dr}{1+r} + \frac{dc_0}{c_0} \quad (\text{A.28})$$

and substituting into (A.26) we get

$$0 = \frac{dc_0}{c_0} \dot{a} \sum_{j=0}^J p_j c_j + s \dot{a} \sum_{j=0}^J j p_j c_j \frac{dr}{1+r}$$

which we can solve out to obtain

$$\frac{dc_0}{c_0} = s \frac{\dot{a} \sum_{j=0}^J p_j j c_j}{\dot{a} \sum_{j=0}^J p_j c_j} \frac{dr}{1+r}$$

<sup>63</sup>This is multiplied by  $1/(1+g)$ , since  $W$  is incoming wealth, which when normalized by GDP growth is  $1/(1+g)$  times smaller than the outgoing wealth from income exceeding consumption in prior periods.

<sup>64</sup>See, for instance, Willis (1988) and Lee (1994).

and hence, plugging back into (A.28),

$$\frac{dc_j}{c_j} = s_j \frac{\dot{a}_{k=0}^J p_k k c_k}{\dot{a}_{k=0}^J p_k c_k} \frac{dr}{1+r} \quad (\text{A.29})$$

i.e. the proportional change in consumption at a given age  $j$  due to the substitution response to an interest rate shock  $\frac{dr}{1+r}$  equals the elasticity of intertemporal substitution  $s$  times the difference between age  $j$  and the average age of consumption. Here, the schedule of consumption by age rotates counterclockwise around the average age of consumption: in response to a rising  $r$ , individuals substitute so that their consumption increases at high ages and decreases at low ages, increasing by more as we get further from the average age.

Plugging (A.29) into (A.27) gives

$$\begin{aligned} dW &= s \frac{1}{1+g} \dot{a}_{j=0}^J p_j c_j \frac{\dot{a}_{k=0}^J p_k k c_k}{\dot{a}_{k=0}^J p_k c_k} \frac{dr}{1+r} \\ &= s \frac{1}{1+g} \dot{a}_{j=0}^J p_j c_j \frac{\dot{a}_{k=0}^J p_k k c_k}{\dot{a}_{k=0}^J p_k c_k} \frac{dr}{1+r} \end{aligned} \quad (\text{A.30})$$

where in the second step we use the fact that  $\dot{a}_{j=0}^J p_j c_j \frac{\dot{a}_{k=0}^J p_k k c_k}{\dot{a}_{k=0}^J p_k c_k} = 0$ . Finally, dividing both sides of (A.30) by  $W$  and multiplying and dividing the right by  $C = \dot{a}_{j=0}^J p_j c_j$ , we get

$$d \log W = s \frac{C}{(1+g)W} \dot{a}_{j=0}^J p_j c_j \frac{\dot{a}_{k=0}^J p_k k c_k}{\dot{a}_{k=0}^J p_k c_k} \frac{dr}{1+r}$$

Now, if we let  $\text{Age}_c$  be a random variable distributed across ages  $j$  with mass proportional to  $p_j c_j$ , then this becomes simply

$$d \log W = s \frac{C}{(1+g)W} \frac{\text{Var Age}_c}{1+r} dr \quad (\text{A.31})$$

which gives us the substitution effect of  $dr$ .

Note that  $\text{Var Age}_c$ , which grows quadratically with the dispersion of consumption across ages, appears in (A.31). This reflects two forces. First, from (A.29) we see that when consumption is further from the average age, it changes by proportionally more in response to a change in  $r$ . Second, financing higher consumption later in life (and correspondingly lower consumption earlier in life) requires holding assets for longer, leading to a larger effect on aggregate assets. Together, these produce the quadratic effect in (A.31).<sup>65</sup>

<sup>65</sup>Although the  $1+g$  and  $1+r$  factors in the denominator of (A.31) are equal in this  $r=g$  special case, we retain them to highlight their separate origin. The  $1+r$  originates with (A.28), since  $d \log(1+r) = dr/(1+r)$ . Meanwhile, the  $1+g$  originates with (A.23), since wealth is measured at the beginning of the period, and yesterday's net saving by a  $1/(1+n)$  smaller generation when productivity was  $1/(1+g)$  as high translates into normalized beginning-of-period wealth today that is  $1/(1+g)$  smaller relative to the

Income effect. Write  $1 + r = (1 + r_{ss})(1 + \tilde{r})$ , so that  $1 + \hat{r} = (1 + \hat{r}_{ss})(1 + \tilde{r})$ . Substituting this into (A.23) and assuming that  $\hat{r}_{ss} = 0$ , we get

$$p_{j+1}a_{j+1} = \frac{1}{1+g}p_j(y_j - c_j) + p_j a_j \tilde{r} + p_j a_j \quad (\text{A.32})$$

Noting that  $a_j \tilde{r}$  enters (A.32) in the same way as  $\frac{1}{1+g}(y_j - c_j)$  (i.e. this extra asset income acts as another form of net income), we can redo the same steps to obtain modified versions of (A.26) and (A.27):

$$0 = \sum_{j=0}^J p_j a_j \tilde{r} + \frac{1}{1+g}(y_j - c_j) \quad (\text{A.33})$$

$$W = \sum_{j=0}^J p_j j \frac{1}{1+g}(c_j - y_j) - a_j \tilde{r} \quad (\text{A.34})$$

Since the interest rate in the Euler equation (A.24) is unchanged, we must have  $dc_j/c_j = \hat{c}$  for some common  $\hat{c}$  across all  $j$ . Totally differentiating, (A.33) thus becomes

$$\frac{1}{1+g} \hat{c} \sum_{j=0}^J p_j c_j = d\tilde{r} \sum_{j=0}^J p_j a_j \quad (\text{A.35})$$

and (A.34) becomes

$$dW = \frac{1}{1+g} \hat{c} \sum_{j=0}^J p_j j c_j - d\tilde{r} \sum_{j=0}^J p_j j a_j \quad (\text{A.36})$$

Dividing both sides of (A.36) by (A.35) (and recalling that  $W = \sum_{j=0}^J p_j a_j$ ), we get

$$\begin{aligned} d \log W &= \frac{\sum_{j=0}^J p_j j c_j}{\sum_{j=0}^J p_j c_j} - \frac{\sum_{j=0}^J p_j j a_j}{\sum_{j=0}^J p_j a_j} d\tilde{r} \\ &= (E \text{Age}_c - E \text{Age}_a) \frac{dr}{1+r} \end{aligned} \quad (\text{A.37})$$

where we define the random variable  $\text{Age}_c$  as before, and analogously  $\text{Age}_a$  as a variable with mass at each age  $j$  proportional to  $p_j a_j$ .

The basic intuition behind (A.37) is the same as in (A.27): total wealth is the gap between the ages at which consumption occurs and the ages at which income is earned. For the income effect, we can think of a rise in  $r$  as an increase in income proportional to assets in each period. Consumption will increase proportionally in every period in response to this extra income; this increased consumption will occur, on average, at the same age as existing consumption. The marginal change in wealth is proportional to the gap between the average age of the marginal consumption ( $E \text{Age}_c$ ) and the average age of the marginal income ( $E \text{Age}_a$ ).

Overall special-case result. Evaluating the semielasticity of wealth-to-GDP with respect to  $r$  in (A.21), noting that the second term is zero because of the Cobb-Douglas assumption, we normalized savings yesterday. (Of course, both factors will tend to be fairly small.)

combine (A.31) and (A.37) to obtain

$$s \frac{C}{(1+g)W} \frac{\text{VarAge}_c}{1+r} + \frac{E\text{Age}_c}{1+r} \frac{E\text{Age}_a}{Z} \quad (\text{A.38})$$

$e_{\text{substitution}}^d$                        $e_{\text{income}}^d$

which is  $\frac{d \log W(r)}{dr}$ .

### B.5.5 General case

**Substitution effect.** For the case  $\hat{r} \neq 0$ , sum both sides of (A.23) from  $j = 0$  to  $j = J$ , making use of the boundary conditions  $a_{J+1} = 0$  and  $a_0 = 0$  and the definition  $W = \sum_{j=0}^J p_j a_j$ , to obtain

$$W = \frac{1}{1+g} p_j (y_j - c_j) + (1+\hat{r})W$$

which can be rearranged as

$$W = \frac{1}{\hat{r}} \sum_{j=0}^J p_j \frac{1}{1+g} (c_j - y_j) \quad (\text{A.39})$$

Applying (A.23), we can obtain the general version of the lifetime budget constraint (A.26)

$$0 = \sum_{j=0}^J p_j (1+\hat{r})^{-j} \frac{1}{1+g} (y_j - c_j) \quad (\text{A.40})$$

The consumption response to a  $r$  shock in the Euler equation is still given by (A.28). Substituting into (A.40), we obtain

$$0 = \frac{dc_0}{c_0} \sum_{j=0}^J p_j (1+\hat{r})^{-j} c_j + \frac{dr}{1+r} s \sum_{j=0}^J p_j (1+\hat{r})^{-j} c_j$$

which we can solve out to obtain

$$\frac{dc_0}{c_0} = -s \frac{\sum_{j=0}^J p_j (1+\hat{r})^{-j} c_j}{\sum_{j=0}^J p_j (1+\hat{r})^{-j} c_j} \frac{dr}{1+r}$$

and hence, plugging back into (A.28),

$$\frac{dc_j}{c_j} = s \sum_{k=0}^j \frac{\sum_{k=0}^J p_k (1+\hat{r})^{-k} c_k}{\sum_{k=0}^J p_k (1+\hat{r})^{-k} c_k} \frac{dr}{1+r} \quad (\text{A.41})$$

which is a slight generalization of (A.29), replacing the average age of consumption  $\frac{\sum_{k=0}^J p_k c_k}{\sum_{k=0}^J p_k c_k}$  with the average age in present value terms discounted by  $\hat{r}$ ,  $\frac{\sum_{k=0}^J p_k (1+\hat{r})^{-k} c_k}{\sum_{k=0}^J p_k (1+\hat{r})^{-k} c_k}$ .

Plugging (A.41) into (A.39), we have

$$\begin{aligned} dW &= \frac{1}{\bar{r}} \sum_{j=0}^J p_j \frac{1}{1+g} s_j \frac{\sum_{k=0}^J k p_k (1+\hat{r})^j c_k}{\sum_{k=0}^J p_k (1+\hat{r})^j c_k} \frac{dr}{1+r} \\ &= s \frac{dr}{1+r} \frac{1}{1+g} \frac{1}{\bar{r}} \sum_{j=0}^J j p_j c_j \frac{\sum_{j=0}^J p_j c_j}{\sum_{j=0}^J p_j (1+\hat{r})^j c_j} A \end{aligned}$$

Dividing both sides by  $W$  and multiplying and dividing the right by  $C = \sum_{j=0}^J p_j c_j$ , we obtain

$$\begin{aligned} d \log W &= \frac{dr}{1+r} \frac{C}{(1+g)W} \frac{1}{\bar{r}} \frac{\sum_{j=0}^J j p_j c_j}{\sum_{j=0}^J p_j c_j} \frac{\sum_{j=0}^J j p_j (1+\hat{r})^j c_j}{\sum_{j=0}^J p_j (1+\hat{r})^j c_j} A \\ &= \frac{dr}{1+r} \frac{C}{(1+g)W} \frac{E \text{Age}_c}{\bar{r}} \frac{E \text{Age}_c^{\text{PV}}}{\bar{r}} \end{aligned} \quad (\text{A.42})$$

where we define  $\text{Age}_c^{\text{PV}}$  as the random variable with probability mass on each age  $j$  proportional to  $p_j (1+\hat{r})^j c_j$ .

**Income effect.** We define  $\bar{r}$  as before, so that  $1+r = (1+r_{ss})(1+\bar{r})$  and  $1+\hat{r} = (1+\hat{r}_{ss})(1+\bar{r})$ , and the budget constraint (A.32) becomes

$$p_{j+1} a_{j+1} = \frac{1}{1+g} p_j (y_j - c_j) + p_j (1+\hat{r}) a_j \bar{r} + (1+\hat{r}) p_j a_j \quad (\text{A.43})$$

Since  $(1+\hat{r}) a_j \bar{r}$  enters into the budget constraint the same way as income net of consumption,  $\frac{1}{1+g}(y_j - c_j)$ , we can write modified versions of (A.40) and (A.39) that incorporate this term:

$$0 = \sum_{j=0}^J p_j (1+\hat{r})^j (1+\hat{r}) a_j \bar{r} + \frac{1}{1+g} (y_j - c_j) \quad (\text{A.44})$$

$$W = \frac{1}{\bar{r}} \sum_{j=0}^J p_j \frac{1}{1+g} (c_j - y_j) (1+\hat{r}) a_j \bar{r} \quad (\text{A.45})$$

Now totally differentiate with respect to  $\bar{r}$ . Since we are not perturbing the  $r$  in the Euler equation, (A.28) implies that  $dc_j/c_j = \hat{c}$  for some common  $\hat{c}$  across all  $j$ . (A.44) can be solved out to obtain

$$\hat{c} = d\bar{r} \frac{(1+\hat{r}) \sum_{j=0}^J p_j (1+\hat{r})^j a_j}{\frac{1}{1+g} \sum_{j=0}^J p_j (1+\hat{r})^j c_j}$$



Plugging this into the totally differentiated (A.45), we obtain

$$\begin{aligned} dW &= \frac{1}{\hat{r}} \sum_{j=0}^J \frac{1}{1+g} p_j c_j \hat{c} \quad d\hat{r} (1+\hat{r}) \sum_{j=0}^J p_j a_j \\ &= d\hat{r} @ \sum_{j=0}^J p_j c_j \quad \frac{(1+\hat{r}) \sum_{j=0}^J p_j (1+\hat{r})^j a_j}{\sum_{j=0}^J p_j (1+\hat{r})^j c_j} \quad (1+\hat{r}) \sum_{j=0}^J p_j a_j A \end{aligned}$$

Dividing both sides by  $W$ , this becomes

$$d \log W = \frac{dr}{1+r} (1+\hat{r}) \frac{C/C^{PV}}{W/W^{PV}} \frac{1}{\hat{r}} = \frac{dr}{1+g} \frac{C/C^{PV}}{W/W^{PV}} \frac{1}{\hat{r}} \quad (\text{A.46})$$

where we identify  $C^{PV} = \sum_{j=0}^J p_j (1+\hat{r})^j c_j$  and  $A^{PV} = \sum_{j=0}^J p_j (1+\hat{r})^j a_j$ , and also write  $d\hat{r} = \frac{dr}{1+r}$ .

**Labor share effect.** In the general, non-Cobb-Douglas case, the  $\frac{\partial \log(w(r)/F(k(r),1))}{\partial r}$  term in (A.45), which is the semielasticity of the labor share with respect to  $r$ , is nonzero.

Normalizing  $L = 1$  and letting  $s_L = w/F(k, 1)$  be the labor share and  $1 - s_L = (r+d)k/F(k, 1)$  be the capital share, we log-differentiate and use the definition of the local elasticity of substitution  $h$  to write

$$d \log s_L = d \log(1 - s_L) = (1 - h) (d \log w - d \log(r + d)) \quad (\text{A.47})$$

Since  $F$  has constant returns to scale, the log change in output price (zero here, since output is the numeraire) must be the share-weighted log change in input prices, so that

$$s_L d \log w + (1 - s_L) d \log(r + d) = 0 \quad (\text{A.48})$$

implying that  $d \log w = -\frac{1-s_L}{s_L} d \log(r + d)$ . Using this and other simplifications, we can rewrite (A.47) as

$$\begin{aligned} \frac{1}{1-s_L} d \log s_L &= (1-h) \frac{1}{s_L} \frac{dr}{r+d} \\ d \log s_L &= (h-1) \frac{1-s_L}{s_L} \frac{dr}{r+d} \end{aligned} \quad (\text{A.49})$$

giving us the semielasticity of the labor share.

**Overall result.** Combining (A.42), (A.46), and (A.49), the semielasticity (A.21) of wealth-to-GDP with respect to  $r$  is

$$s \frac{1}{1+r} \frac{C}{(1+g)W} \frac{E A g_c}{\hat{r}} \frac{E A g_c^{PV}}{\hat{r}} + \frac{1}{1+g} \frac{C/C^{PV}}{W/W^{PV}} \frac{1}{\hat{r}} + (h-1) \frac{(1-s_L)/s_L}{\frac{r+d}{z}} \quad (\text{A.50})$$

$e_{\text{substitution}}^d$                        $e_{\text{income}}^d$                        $e_{\text{laborshare}}^d$

which is our main result.

Continuity in the  $\hat{r} \rightarrow 0$  limit. Taking the limit of  $\frac{EAge_c - EAge_c^{PV}}{\hat{r}}$  as  $\hat{r} \rightarrow 0$  using L'Hospital's rule, we get:

$$\lim_{\hat{r} \rightarrow 0} \frac{1}{\hat{r}} \frac{\sum_{j=0}^J p_j c_j}{\sum_{j=0}^J p_j c_j} \frac{\sum_{j=0}^J p_j (1 + \hat{r})^j c_j}{\sum_{j=0}^J p_j (1 + \hat{r})^j c_j} A = EAge_c \frac{\sum_{j=0}^J j^2 p_j c_j}{\sum_{j=0}^J j p_j c_j} \frac{\sum_{j=0}^J j p_j c_j}{\sum_{j=0}^J p_j c_j} A$$

$$= EAge_c \left( \frac{EAge_c^2}{EAge_c} - EAge_c \right) = EAge_c^2 - (EAge_c)^2 = \text{Var}Age_c$$

which makes the  $e_{\text{substitution}}^d$  term in (A.50) identical to (A.38).

Similarly, taking the limit of  $\frac{C/C^{PV} - W/W^{PV}}{\hat{r}}$  as  $\hat{r} \rightarrow 0$  using L'Hospital's rule, we get:

$$\lim_{\hat{r} \rightarrow 0} \frac{1}{\hat{r}} \frac{\sum_{j=0}^J p_j c_j}{\sum_{j=0}^J p_j c_j} \frac{\sum_{j=0}^J p_j (1 + \hat{r})^j c_j}{\sum_{j=0}^J p_j (1 + \hat{r})^j c_j} A = \frac{\sum_{j=0}^J j p_j c_j}{\sum_{j=0}^J p_j c_j} \frac{\sum_{j=0}^J j p_j a_j}{\sum_{j=0}^J p_j a_j} = EAge_c - EAge_a$$

which, when also using the fact that  $\hat{r} = 0$  implies  $1 + g = 1 + r$ , makes the  $e_{\text{income}}^d$  term in (A.50) identical to (A.38).

## C Appendix to Section 3

### C.1 Data sources

**Demographics.** Our population data and projections comes from the 2019 UN World Population Prospects.<sup>66</sup> We gather data between 1950 and 2100 on total number of births, number of births by age-group of the mother, population by 5-year age groups, and mortality rates by 5-year age groups. We interpolate to construct population distributions  $N_{jt}$  and mortality rates  $f_{jt}$  in every country, every year, and for every age. We compute total population as  $N_t = \sum_j N_{jt}$ , population distributions as  $p_{jt} = N_{jt} / N_t$ , and population growth rates as  $1 + n_t = N_t / N_{t-1}$ . Finally, we compute the number of migrants by age  $M_{jt}$  as the residual of the population law of motion

$$N_{jt} = (N_{j,t-1} + M_{j,t-1}) f_{j,t-1}$$

**Age-income profiles.** We use the LIS to construct the base-year age-income profiles for all the countries we consider. For Australia, the LIS is based on the Survey of Income and Housing (SIH) and the Household Expenditure Survey (HES), for Austria on the Survey on Income and Living Conditions (SILC), for Canada on the Canadian Income Survey (CIS), for China on the Chinese Household Income Survey (CHIP), for Denmark on the Law Model (based on administrative records), for Estonia on the Estonian Social Survey (ESS) and the Survey on Income and Living Conditions (SILC), for Finland on the Income Distribution Survey (IDS) and the Survey on Income and Living Conditions (SILC), for France on the Household Budget Survey (BdF), for Germany on the German Socio-Economic Panel (GSOEP), for Greece on the Survey of Income and Living Conditions (SILC), for Hungary on the Tárki Household Monitor Survey, for India on the India Human Development Survey (IHDS), for Ireland on the Survey on Income and Living Conditions (SILC), for Italy on the Survey of Household Income and Wealth (SHIW), for Japan on the Japan

<sup>66</sup><https://population.un.org/wpp/>

Household Panel Survey (JHPS), for Luxembourg on the Socio-economic Panel “Living in Luxembourg” (PSELL III) and the Survey on Income and Living Conditions (SILC), for Netherlands on the Survey on Income and Living Conditions (SILC), for Norway on the Household Income Statistics, for Poland on the Household Budget Survey, for Slovakia on the Survey of Income and Living Conditions (SILC), for Slovenia on the Household Budget Survey (HBS), for Estonia on the Survey on Income and Living Conditions (SILC), for Sweden on the Household Income Survey (HINK/HEK), and for the United Kingdom on the Family Resources Survey (FRS).

**Age-wealth profiles.** Our wealth data for the United States comes from the 2016 Survey of Consumer Finance. We gather data from other countries as follows. First, we take data from the Luxembourg Wealth Study (LWS)<sup>67</sup> for Australia in 2010, Canada in 2012, Germany in 2012, United Kingdom in 2011, Italy in 2010, and Sweden in 2005. For Australia the LWS is based on the Survey of Income and Housing (SIH) and the Household Expenditure Survey (HES), for Canada on the Survey of Financial Securities (SFS), for Germany on the German Socio-Economic Panel (GSOEP), for Italy on the Survey of Household Income and Wealth (SHIW), for Sweden on the Household Income Survey (HINK/HEK), and for United Kingdom on the Wealth and Assets Survey (WAS). We rescale the survey weights such that they sum up to the correct number of households according to, respectively, the Australian Bureau of Statistics, Statistics Canada, Statistisches Bundesamt, the Office for National Statistics, the Istituto Nazionale di Statistica, and the United Nations Economic Commission for Europe (UNECE). Next, we use the Household Finance and Consumption Survey (HFCS)<sup>68</sup> for Austria in 2010, Belgium in 2010, Estonia in 2014, Spain in 2010, Finland in 2010, France in 2010, Greece in 2010, Hungary in 2014, Ireland in 2014, Luxembourg in 2014, Netherlands in 2010, Poland in 2014, Slovenia in 2014, and Slovakia in 2014. For China, we rely on the 2013 China Household Finance Survey (CHFS).<sup>69</sup> For India, we use the National Sample Survey (NSS).<sup>70</sup> For Japan, we construct a measure of total wealth by age of household head from Table 4 of the 2009 National Survey of Family Income and Expenditure (NFSIE) available on the online portal of Japanese Government Statistics<sup>71</sup>. This table provides average net worth and total number of households by age groups for single person households and households with two or more members, which we aggregate to obtain total household net worth by age group. For Denmark, we use the 2014 table “Assets and liabilities per person by type of components, sex, age and time” produced by Statistics Denmark that provides a measure of average net wealth per person by age group produced from tax data.

**Aggregation.** We cross-check the wealth data aggregated from the household survey with the aggregate wealth-to-GDP ratio provided by the WID or the OECD. Table A.1 provides details on the source of both survey and aggregate data, as well as the wealth-to-GDP ratio computed from the survey, compared to the official statistic.

## C.2 Robustness

In this section, we show that our results are robust to some of our main assumptions behind the calculation of compositional effects. In the interest of space, we focus here on the United States;

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<sup>67</sup><https://www.lisdatacenter.org/our-data/lws-database/>

<sup>68</sup>[https://www.ecb.europa.eu/stats/ecb\\_surveys/hfcs/](https://www.ecb.europa.eu/stats/ecb_surveys/hfcs/)

<sup>69</sup><http://www.chfsdata.org/>

<sup>70</sup><http://microdata.gov.in>

<sup>71</sup><https://www.e-stat.go.jp>

Table A.1: Wealth-to-GDP ratios from survey data and aggregate data

Country	Wealth survey data			Aggregate data		
	Year	Source	$\frac{W^c}{Y^c}$	Year	Source	$\frac{W^c}{Y^c}$
AUS	2014	LWS	3.59	2016	WID	5.09
AUT	2014	HFCS	2.79	2016	OECD	3.90
BEL	2014	HFCS	3.84	2016	OECD	5.74
CAN	2016	LWS	6.98	2016	WID	4.63
CHN	2013	CHFS	3.27	2016	WID	4.20
DEU	2017	LWS	3.8	2016	WID	3.64
DNK	2014	SD	2.54	2016	WID	3.42
ESP	2014	HFCS	4.96	2016	WID	5.33
EST	2014	HFCS	2.78	2016	OECD	2.64
FIN	2014	HFCS	2.33	2016	WID	2.78
FRA	2014	HFCS	3.30	2016	WID	4.85
GBR	2011	LWS	4.01	2016	WID	5.35
GRC	2014	HFCS	2.65	2016	WID	4.25
HUN	2014	HFCS	1.84	2016	OECD	2.19
IND	2013	NSS	4.01	2016	-	-
IRL	2014	HFCS	3.39	2016	CBI	2.32
ITA	2016	LWS	3.35	2016	WID	5.83
JPN	2009	NSFIE	6.11	2016	WID	4.85
LUX	2014	HFCS	3.80	2016	OECD	3.92
NLD	2014	HFCS	1.80	2016	WID	3.92
POL	2014	HFCS	3.31	2016	OECD	1.50
SVK	2014	HFCS	1.80	2016	OECD	2.17
SVN	2014	HFCS	3.11	2016	OECD	2.82
SWE	2005	LWS	2.00	2016	WID	3.81
USA	2016	SCF	4.38	2016	WID	4.28

Notes: This table summarizes our sources of wealth survey data and aggregate data. Abbreviations are described in the text. The survey-based wealth to GDP ratio  $W^c/Y^c$  is computed by aggregating household wealth using survey weights and dividing by GDP per household from the national accounts.

conclusions are similar when repeating this exercise in other countries.

**Alternative allocation of household wealth across individual members.** All our surveys measure wealth at the household level. In the main text, we obtain individual wealth by splitting up all assets equally between all members of the household that are at least as old as the head or spouse. The orange line in [Figure A.3](#), labeled "baseline", reproduces the projection from the United States under the main fertility scenario (cf [figures 2 and 5](#)). The red line shows that allocating all household wealth to the head increases the compositional effect a little, since heads tend to be older on average; the grey line shows that allocating all wealth equally to head as spouse, as in [Poterba \(2001\)](#), or equally to all household members aged 20 or older. This delivers results extremely close to our baseline.

**Constructing compositional effects at the household level.** All our exercises in the main text of [section 3.1](#), as well as the alternative considered in the previous paragraph, are conducted at the individual level. To gauge the importance of the household vs individual distinction, here we calculate compositional effects at the household level instead.

We first obtain the age-wealth and labor income profiles at the household level, summing the pre-tax labor income of each household member. To convert the age distribution of the population over individuals to an age distribution over households, we use the PSID to estimate a mapping that gives, for each age  $j$ , the age of the household head than an average individual of age  $j$  lives with.

With this data in hand, we recompute the compositional effect  $D^{\text{comp}}$ . [Figure A.3](#) reports the projected change in  $W/Y$  from this exercise under the baseline fertility scenarios. The dashed line reproduces the central individual-level compositional effect from the main text. Overall, the timing of the projected changes in  $W/Y$  change slightly, but the overall magnitude remains close.

**Alternative choice of base year profiles.** [Table A.2 and A.3](#) explores how the magnitude of the compositional effects  $D^{\text{comp}}$  changes when we change the base year 0 we use to construct the age profiles  $a_{j0}$  and  $h_{j0}$  in [equation \(10\)](#).

In the last row and column, labeled "DH", we use the age effects extracted from a time-age-cohort decomposition in the style of [Hall \(1968\)](#) and [Deaton \(1997\)](#), imposing that all growth loads on time effects. It is important to load growth on time effects to recover the age profiles that are the correct input into [Proposition 1](#).

Using earlier data for age-wealth profiles tends to imply smaller effects, since the age-wealth profile has steepened over time. (The 1977 data stands out as an outlier implying especially small effects; the age-wealth profile in that year declined much more rapidly at higher ages.) Using earlier data for age-labor income profiles tends to imply slightly larger effects, since the hump-shape in the age-labor income profile has moved to the right over time as generations retire later. Overall, using earlier data for both profiles implies mildly smaller effects. In contrast, using the age effects from our time-age-cohort decomposition ("DH-t") implies a slightly larger compositional effect.

### C.3 Additional results for [section 3.1](#)

Historical predicted change in  $W/Y$  from composition effects vs actual change in  $W/Y$ . [Table A.4](#) contrasts, for a range of countries for which the World Inequality Database contains a sufficiently long time series of measured wealth-to-GDP ratios, the measured change in the log of

Table A.2: Sensitivity of predicted change in US log W/ Y to choice of base year

Panel A. Predicted change in log W/ Y from composition between 2016 and 2100													
	h <sub>j</sub> year												
a <sub>j</sub> year	1974	1979	1986	1991	1994	1997	2000	2004	2007	2010	2013	2016	DH
1989	26.5	26.1	25.8	25.5	25.5	25.2	25.2	24.7	24.3	24.0	23.6	23.5	27.6
1992	22.5	22.1	21.8	21.5	21.5	21.2	21.2	20.7	20.3	20.0	19.6	19.4	23.6
1995	25.5	25.1	24.8	24.6	24.5	24.2	24.2	23.7	23.3	23.0	22.6	22.5	26.7
1998	22.7	22.4	22.0	21.8	21.7	21.4	21.4	20.9	20.5	20.2	19.8	19.7	23.9
2001	22.5	22.1	21.8	21.5	21.4	21.2	21.1	20.7	20.2	20.0	19.5	19.4	23.6
2004	25.7	25.3	25.0	24.7	24.7	24.4	24.4	23.9	23.5	23.2	22.7	22.6	26.8
2007	24.5	24.1	23.8	23.5	23.5	23.2	23.1	22.7	22.3	22.0	21.5	21.4	25.6
2010	28.7	28.4	28.1	27.8	27.7	27.5	27.4	26.9	26.5	26.3	25.8	25.7	29.9
2013	28.2	27.9	27.5	27.3	27.2	26.9	26.9	26.4	26.0	25.7	25.3	25.2	29.4
2016	30.9	30.5	30.2	29.9	29.9	29.6	29.5	29.1	28.7	28.4	27.9	27.8	32.0
DH-t	28.0	27.7	27.3	27.1	27.0	26.7	26.7	26.2	25.8	25.5	25.1	25.0	29.2

Panel B. Predicted change in log W/ Y from composition between 1950 and 2016													
	h <sub>j</sub> year												
a <sub>j</sub> year	1974	1979	1986	1991	1994	1997	2000	2004	2007	2010	2013	2016	DH
1989	22.8	22.5	22.3	21.7	21.2	20.7	21.1	20.1	19.8	19.1	19.0	19.3	29.3
1992	22.7	22.4	22.2	21.6	21.1	20.6	21.0	20.1	19.8	19.0	18.9	19.2	29.2
1995	24.3	24.0	23.8	23.2	22.7	22.2	22.6	21.7	21.4	20.6	20.5	20.8	30.8
1998	23.6	23.3	23.1	22.6	22.0	21.5	21.9	21.0	20.7	20.0	19.9	20.1	30.2
2001	23.8	23.5	23.3	22.8	22.2	21.8	22.2	21.2	20.9	20.2	20.1	20.3	30.4
2004	25.4	25.1	24.9	24.3	23.8	23.3	23.7	22.8	22.5	21.7	21.6	21.9	31.9
2007	24.8	24.5	24.3	23.8	23.2	22.7	23.1	22.2	21.9	21.2	21.1	21.3	31.4
2010	27.9	27.6	27.4	26.9	26.3	25.8	26.2	25.3	25.0	24.3	24.2	24.4	34.5
2013	26.7	26.4	26.2	25.6	25.1	24.6	25.0	24.0	23.7	23.0	22.9	23.2	33.2
2016	28.1	27.9	27.6	27.1	26.5	26.1	26.5	25.5	25.2	24.5	24.4	24.6	34.7
DH-t	29.2	29.0	28.8	28.2	27.6	27.2	27.6	26.6	26.3	25.6	25.5	25.8	35.8

Notes: This table reports the US  $D^{\text{comp}}$ , the predicted change in log W/ Y from compositional effects as defined in equation (10), for alternative base years of the age-wealth and the age-labor income profiles, reported in percent. Panel A considers our main period of interest 2016 to 2100, and panel B considers 1950 to 2016. Every column corresponds to an alternative base year for the age-labor income profile, and every row to an alternative base year for the age-wealth profile. The last row and column correspond to the cases where we use the average age effect from a time-age-cohort decomposition on the 1989–2016 SCF data, with all growth loading on time effects (DH, for “Deaton-Hall”).

Table A.3: Sensitivity of predicted change in US log W/ Y to choice of earlier base year

Panel A. Predicted change in log W/ Y from composition between 2016 and 2100													
$a_j$ year	$h_j$ year												DH
	1974	1979	1986	1991	1994	1997	2000	2004	2007	2010	2013	2016	
1958	20.8	20.4	20.1	19.8	19.8	19.5	19.5	19.0	18.6	18.3	17.8	17.7	21.9
1959	17.0	16.6	16.3	16.0	16.0	15.7	15.7	15.2	14.8	14.5	14.1	14.0	18.2
1960	17.5	17.2	16.9	16.6	16.5	16.3	16.2	15.7	15.3	15.0	14.6	14.5	18.7
1962	17.4	17.0	16.7	16.4	16.3	16.1	16.0	15.6	15.1	14.9	14.4	14.3	18.5
1965	19.0	18.6	18.3	18.0	18.0	17.7	17.6	17.2	16.8	16.5	16.0	15.9	20.1
1967	21.5	21.1	20.8	20.5	20.5	20.2	20.1	19.7	19.3	19.0	18.5	18.4	22.6
1968	19.5	19.1	18.8	18.6	18.5	18.2	18.2	17.7	17.3	17.0	16.6	16.5	20.7
1969	19.8	19.4	19.1	18.9	18.8	18.5	18.5	18.0	17.6	17.3	16.9	16.8	21.0
1970	24.0	23.6	23.3	23.0	23.0	22.7	22.6	22.2	21.8	21.5	21.0	20.9	25.1
1977	11.5	11.1	10.8	10.5	10.5	10.2	10.2	9.7	9.3	9.0	8.6	8.5	12.7
1983	23.8	23.5	23.2	22.9	22.8	22.6	22.5	22.0	21.6	21.3	20.9	20.8	25.0
2016	30.9	30.5	30.2	29.9	29.9	29.6	29.5	29.1	28.7	28.4	27.9	27.8	32.0

Panel B. Predicted change in log W/ Y from composition between 1950 and 2016													
$a_j$ year	$h_j$ year												DH
	1974	1979	1986	1991	1994	1997	2000	2004	2007	2010	2013	2016	
1958	12.7	12.4	12.2	11.6	11.1	10.6	11.0	10.1	9.8	9.0	8.9	9.2	19.3
1959	15.9	15.6	15.4	14.8	14.3	13.8	14.2	13.3	13.0	12.2	12.1	12.4	22.5
1960	17.5	17.2	17.0	16.5	15.9	15.4	15.8	14.9	14.6	13.9	13.8	14.0	24.1
1962	18.0	17.7	17.5	16.9	16.4	15.9	16.3	15.4	15.1	14.3	14.2	14.5	24.5
1965	14.7	14.4	14.2	13.6	13.1	12.6	13.0	12.1	11.8	11.0	10.9	11.2	21.2
1967	18.0	17.7	17.5	16.9	16.4	15.9	16.3	15.4	15.1	14.3	14.2	14.5	24.5
1968	17.2	16.9	16.7	16.2	15.6	15.1	15.5	14.6	14.3	13.6	13.4	13.7	23.8
1969	18.0	17.7	17.5	17.0	16.4	15.9	16.3	15.4	15.1	14.4	14.3	14.5	24.6
1970	21.9	21.6	21.4	20.8	20.2	19.8	20.2	19.2	18.9	18.2	18.1	18.4	28.4
1977	10.6	10.3	10.1	9.5	9.0	8.5	8.9	8.0	7.7	6.9	6.8	7.1	17.1
1983	21.4	21.1	20.9	20.4	19.8	19.3	19.7	18.8	18.5	17.8	17.6	17.9	28.0
2016	28.1	27.9	27.6	27.1	26.5	26.1	26.5	25.5	25.2	24.5	24.4	24.6	34.7

Notes: This table reports the US  $D^{comp}$ , the predicted change in log W/ Y from compositional effects as defined in equation (10), for alternative base years of the age-wealth and the age-labor income profiles, reported in percent. Compared to table A.2, this table considers earlier SCF waves for the age-wealth profile, as constructed by Kuhn et al. (2020). Panel A considers our main period of interest 2016 to 2100, and panel B considers 1950 to 2016. Every column corresponds to an alternative base year for the age-labor income profile, and every row to an alternative base year for the age-wealth profile. The last column corresponds to the case where we use the average age effect from a time-age-cohort decomposition on the 1989–2016 SCF data, with all growth loading on time effects (DH, for “Deaton-Hall”).

Figure A.3: Predicted change in USW/ Y from composition: alternative assumptions

Notes: This figure depicts the evolution of the predicted change in the wealth-to-GDP ratio from the compositional effect, calculated using equation (21) from  $t = 1950$  to 2100. The orange line corresponds to our baseline case, where the wealth of households is allocated equally to all members at least as old as the head or the spouse. The red line shows the outcome when wealth is allocated to the head of household only, the gray line to the head and the spouse equally, and the green line to all members aged 20 or more. The blue line presents the outcome when the analysis is conducted at the household-level rather than at the individual level.

$W/ Y$  (labelled "Data") relative to the compositional effect  $D_t^{\text{comp}}$  (labelled "Comp"). The latter is constructed from equation 10 using age profiles from the baseline year interacted with the actual change in population distributions over the period reported. Both columns are multiplied by 100 to be interpretable as percentage points. The compositional effect predicts an increase in  $W/ Y$  in every country, consistent with what occurred. For countries like the United States and the Netherlands, the magnitudes also line up closely; for Spain, the compositional effect overpredicts the historical magnitude, while for most other countries the historical increase in  $W/ Y$  is greater than the compositional effect alone would predict. If demographics was the only force driving wealth-to-GDP ratios then our theory suggests that the rise in  $W/ Y$  should be less than what is predicted by the compositional effect due to the endogenous response of asset returns; the fact that many countries experienced larger increases suggests that other forces, such as declining productivity growth, have also been at play.

Role of heterogeneity in demographic change vs age profiles. Figure A.4 presents the predicted change in  $W/ Y$  between 2016 and 2100 from the compositional effect and isolates the contributions from demographic forces and from the age-profiles. Panel A repeats the results from section 3.1, ranking countries from lowest to highest compositional effect. It also presents the results under the two UN fertility scenarios. To isolate the contribution from demographic forces, panel B computes the compositional effect where age-profiles in all countries are identical to the



A. Baseline and low/high fertility scenarios

B. At common age profiles

C. At common demographic change

Figure A.4: Predicted change in  $W/Y$  from composition between 2016 and 2100

Notes: Panel A presents the change in  $W/Y$  between 2016 and 2100 from equation (21) as well as its value using the low fertility (circles) and high fertility (squares) scenarios. Panel B does this calculation again, assuming that all countries have US age profiles of assets and income. Panel C does this calculation again, assuming all countries have the US age distribution in every year.

Table A.4: Historical change in log (W/ Y) vs predicted change from  $D^{\text{comp}}$  (in log %)

Country	Period	Data	Comp.
AUS	1960-2016	59.8	15.6
CAN	1970-2016	82.6	19.2
CHN	1978-2016	140.9	16.8
DEU	1950-2016	67.4	23.7
DNK	1973-2016	80.2	13.8
ESP	1950-2016	19.1	27.6
FIN	2011-2016	9.2	5.5
FRA	1950-2016	109.3	21.4
GBR	1950-2016	37.5	18.9
GRC	1997-2016	17.3	8.7
IND	1950-2016	23.2	10.9
ITA	1966-2016	108.8	23.5
JPN	1970-2016	66.0	42.5
NLD	1997-2016	23.4	21.1
SWE	1950-2016	48.8	19.6
USA	1950-2016	31.6	27.5

US pro les. To isolate the contribution from the pro les, panel C computes the compositional effect where population distributions of the US are used in every country. Panels B and C show that both the shapes of the pro les and the changes in population distributions matter to the compositional effect, but that the demographic forces play a much more important role in generating shift-shares that are high and heterogeneous across countries.

#### C.4 Additional results for sections 3.2 and 3.3

Age pro les of consumption and assets. Figure A.5 presents the age distributions of consumption (orange lines) and asset holdings (red lines), constructed using the procedure described in section 3.2. The consumption pro le is backed out of the asset pro le and the pro le of net income. Net income includes all taxes and transfers; since this measure is not available in most surveys, we back it out of aggregate information on taxes and transfers. In practice, we use net income from our quantitative model of section 4, which is constructed using that information for each country.

Applying equation (15) at each point in time to predict NFAs. Figure A.6 reproduces Figure 7, but we applying equation (15) at each point in time to predict NFAs. Speci cally, we apply equation

$$\log \left( 1 + \frac{NFA_t^c / Y_t - NFA_0^c / Y_0}{W_0^c / Y_0^c} \right) = D_t^{\text{comp}c} \bar{D}_t^{\text{comp}} + e^{c,d} + e^{c,s} \quad \bar{e}^d + \bar{e}^s \quad (r_t - r_0) \quad (\text{A.51})$$

Figure A.5: Distribution of the ages of consumption and wealth in each country.

Notes: This figure presents the age distributions of consumption (orange lines) and asset holdings (red lines). The dashed vertical lines depict the average ages of consumption and asset holdings.

Figure A.6: Using a dynamic version of equation (15) to project NFAs

Notes: This reproduces figure 7, but uses (A.51)–(A.52), rather than  $D_t^{\text{comp}c} = \bar{D}_t^{\text{comp}}$ , to project  $D_t^{\text{NFA}c}$ .

where  $r_t = r_0$  is, in turn, calculated by applying equation (13) at each point in time,

$$r_t = r_0 + \frac{1}{\bar{e}^d + \bar{e}^s} \bar{D}_t^{\text{comp}} \quad (\text{A.52})$$

and, in equations (A.51)–(A.52), we take  $\bar{e}^d$  and  $\bar{e}^s$  to be the steady state elasticities calculated using our sufficient statistics.<sup>72</sup>

The main findings from figure A.6 are unchanged relative to those from figure 7, indicating that the interest rate adjustment term does not play a major role when it comes to forecasting NFAs. This is because this interest adjustment only matters to the extent that elasticities of supply and demand differ across countries, and the heterogeneity we calculate from our sufficient statistics is relatively limited.

## D Appendix to Section 4

### D.1 Full model setup

Here, we describe the model in section 4. We first describe the full model for one country, omitting the country superscript  $c$ , and define a small open economy equilibrium for a fixed sequence  $\{r_t\}_t$ . The world equilibrium is defined as a sequence  $\{r_t\}_t$  that clears the global asset market.

**Demographics.** The demographics are given by a sequence of births  $\{N_{0t}g_{t-1}\}$ , a sequence of age- and time-specific survival rates  $\{f_{jt}g_{t-1}\}$  for individuals between age  $j$  and  $j+1$ , a sequence of net migration levels  $\{M_{jt}g_{t-1}\}_{t,0 \leq j \leq T-1}$ , as well as an initial number of agents by age  $N_{j, -1}$ . The

<sup>72</sup>The exact first-order approximation involves a sequence-space Jacobian matrix (Auclert, Bardóczy, Rognlie and Straub 2021). In practice, however, we are unaware of a sufficient statistic expression for the Jacobian that underlies  $\bar{e}^d$ . Figure 8 shows that the approximation in (A.51)–(A.52) works fairly well in the context of our structural model.

assumption that demographic variables start at  $t = -1$  is made for technical reasons; it allows us to correctly account for migration and bequests received at time  $t = 0$ . Given these parameters, the population variables evolve according to the exogenous  $N_{0t}$  and

$$N_{jt} = (N_{j-1,t-1} + M_{j-1,t-1})f_{j-1,t-1}, \quad \forall t \geq 0, j > 0. \quad (\text{A.53})$$

As in section 2, we write  $N_t = \sum_j N_{jt}$  for the total population at time  $t$ , and  $p_{jt} = \frac{N_{jt}}{N_t}$  for the age distribution of the population.

**Agents' problem.** The basic setup is the same as in section 2, with heterogeneous individuals facing idiosyncratic income risk. We restrict the income process so that effective labor supply  $\tilde{y}_{jt}$  is the product of a deterministic term  $\tilde{y}_j$  that varies across ages, a fixed effect  $q$ , and a transitory component  $e$ , where both the fixed effect and the transitory component have a mean of 1. The log transitory component follows a finite-state Markov process with a transition matrix across years  $P^e(e|e')$  from  $e'$  to  $e$ , calibrated to have a persistence  $c_e$  and a standard deviation  $u_e$ , while the log permanent component follows a discrete Markov process across generations with a transition matrix  $P^q(q|q')$  from  $q'$  to  $q$  calibrated to have a persistence  $c_q$  and a standard deviation  $u_q$ . The processes are independent, and we write  $p^e(e)$  and  $p^q(q)$  for the corresponding stationary probability mass functions.<sup>73</sup>

We assume that individuals become economically active at age  $J^w$ , so that labor income at age  $j$  at time  $t$  is  $w_t(1 - r_{jt})q\tilde{y}_{jt}$ , where  $w_t$  is the wage per efficiency unit as in section 2, and  $r_{jt} \in [0, 1]$  is a parameter of the retirement system indicating the fraction of labor that households of age  $j$  are allowed to supply at time  $t$ . After retirement, agents receive social security payments  $w_t r_{jt} q d_t$  in proportion to their permanent type, where  $d_t$  encodes a time-varying social security replacement rate.

The state for an individual at age  $j$  and time  $t$  is given by the fixed effect  $q$ , the transitory effect  $e$ , and asset holdings  $a$ , and their value function is given by

$$V_{jt}(q, e, a) = \max_{c, a^0} \frac{c^{1-\frac{1}{s}}}{1-\frac{1}{s}} + UZ_t^n \frac{1-\frac{1}{s}}{s} (1 - f_{jt}) \frac{(a^0)^{1-n}}{1-n} + f_{jt} \frac{b_{j+1}}{b_j} E V_{j+1,t+1}(q, e^0, a^0) \\ c + a^0 - w_t q (1 - r_{jt})(1 - t_t) \tilde{y}_{jt} e + r_{jt} d_t + (1 + r_t)[a + b_{jt}^r(q)] \\ \bar{a} Z_t a^0, \quad (\text{A.54})$$

which determines the decision function  $c = c_{jt}(q, e, a)$  and  $a^0 = a_{j+1,t+1}(q, e, a)$  for consumption and next-period assets.

The term  $\frac{c^{1-\frac{1}{s}}}{1-\frac{1}{s}}$  represents the flow utility of consumption, and  $UZ_t^n \frac{1-\frac{1}{s}}{s} (1 - f_{jt}) \frac{(a^0)^{1-n}}{1-n}$  represents the utility from giving bequests  $a^0$ . The bequest utility is scaled by mortality risk  $1 - f_{jt}$ , since agents only give bequests if they die, and  $n - \frac{1}{s}$  captures potential non-homotheticities in bequests, which has been shown to generate more realistic levels of wealth inequality (De Nardi, 2004). The scaling factor  $Z_t^n \frac{1-\frac{1}{s}}{s}$  ensures balanced growth in spite of this non-homotheticity. The term  $b_{jt}^r(q)$  represents bequests received, and is allowed to vary according to the agent's permanent type.

<sup>73</sup>Discrete processes are used to facilitate notation. The calibration to the persistence and standard deviation is done using Tauchen's method applied to a Gaussian AR(1) process with a given persistence, standard deviation, and mean.

State distribution. To determine the evolution of states, we assume that the distribution of individuals across  $q$  and  $e$  is in the stationary distribution for all ages and times, as well as for arriving and leaving migrants. This implies that the joint distribution across  $(q, e, a)$  is fully characterized by

$$H_{jt}(ajq, e) = P(a_{jt} | ajq, e),$$

where  $H_{jt}$  is the conditional probability distribution of assets given  $q$  and  $e$ .<sup>74</sup>

Over time, the distribution evolves according to

$$H_{j+1,t+1}(ajq, e) = \int_e \frac{P^e(e|e^0)p^e(e^0)}{p^e(e)} \int_{a^0}^Z I(a_{j+1,t+1}(a^0, q, e) > a) dH_{j,t}(a^0|q, e) \quad \forall j > J^w, \quad (A.55)$$

where  $a_{j+1,t+1}$  is the decision function for assets implied by the agents' problem (A.54). Note that (A.55) implicitly assumes that death is independent of asset holdings and that migrants have the same distribution of assets as residents. At time zero, there is an exogenous distribution of assets  $H_{j0}(j|q, e)$  for each age group. As a boundary condition, we assume that individuals do not have any assets before working life starts:

$$H_{j,t}(ajq, e) = I(a = 0) \quad \forall q, e, j \in J^w, 0 \leq t, \quad (A.56)$$

where  $I$  is the indicator function.

Bequest distribution. We model partial intergenerational wealth persistence by assuming that all bequests from individuals of type  $q$  are pooled and distributed across the types  $q^0$  of survivors in accordance with the intergenerational transmission of types. Formally, the total amount of bequests received by agents of type  $q^0$  of age  $j$  at time  $t$  satisfies

$$N_{jt} b_{jt}^r(q) = F_j \int_q \frac{P^q(q|q^0)p^q(q^0)}{p^q(q)} \int_{k=0}^T [N_{k,t-1} + M_{k,t-1}](1 - f_{k,t-1}) \int_e \int_a p^e(e) \int_a^Z adH_{kt}(ajq^0, e) \quad (A.57)$$

Here,  $\int_k [N_{k,t-1} + M_{k,t-1}](1 - f_{k,t-1}) \int_e \int_a p^e(e) \int_a^Z adH_{kt}(ajq^0, e)$  captures the total amount of bequests given by individuals of type  $q^0$ . The timing is that migrants arrive before the death event and that interest rate accrues after the death event. A share  $\frac{P^q(q|q^0)p^q(q^0)}{p^q(q)}$  of these bequests is given to agents of type  $q$ , capturing partial intergenerational transmission by using the probability that an agent of type  $q$  has a parent of type  $q^0$ .

Note that an aging population alters the relative number of agents that give relative to the number of agents that receive bequests, which ceteris paribus increases bequest sizes. The migrants are included, assuming that migrants have the same mortality as the overall population, and that migrants who plan to arrive at  $t$  but die between  $t-1$  and  $t$  augment the bequest pool in the receiving country.

<sup>74</sup>Formally, given  $H_{jt}$ , the joint distribution function  $\tilde{H}_{jt}$  of  $(q, e, a)$  can be written  $\tilde{H}_{jt}(q, e, a) = \int_{q^0} \int_{e^0} p^q(q^0)p^e(e^0)H_{jt}(ajq^0, e^0)$ .

Aggregation. Given the decision functions  $c_{jt}$  and  $a_{j+1,t+1}$  and the distribution across states, aggregate consumption and assets satisfy

$$\begin{aligned} W_t &= \sum_{j=0}^J N_{jt} \int_{e,q} \dot{a} p^e(e) p^q(q) \int_a^Z [a + b_{jt}^r(q)] dH_{jt}(a; q, e) \\ C_t &= \sum_{j=0}^J N_{jt} \int_e \dot{a} p^e(e) \int_a^Z c_{jt}(q, e, a) dH_{jt}(ajq, e). \end{aligned} \quad (A.58)$$

Note that bequests received are included in the definition of today's ingoing assets.

Production. As in section 2, markets are competitive, there are no adjustment costs in capital, and there is labor-augmenting growth at a constant rate  $g$ . Production is given by a CES aggregate production function. We obtain the following equations:

$$Y_t = F(K_t, Z_t L_t) = a K_t^{\frac{h-1}{h}} + (1-a) [Z_t L_t]^{\frac{h-1}{h}} \quad (A.59)$$

$$Z_t = (1+g)^t Z_0 \quad (A.60)$$

$$r_t = F_K(K_t, Z_t L_t) - d \quad (A.61)$$

$$w_t = Z_t F_L(K_t, Z_t L_t) \quad (A.62)$$

$$K_{t+1} = (1-d)K_t + I_t \quad (A.63)$$

$$L_t = \sum_{j=0}^J N_{jt} (1-r_{jt})^{-j}, \quad (A.64)$$

where the last line uses that  $E q e = 1$  to obtain that average effective labor supply is  $\sum_j$  of individuals of age  $j$ .

Government. The government purchases  $G_t$  goods and sets the retirement policy  $r_{jt}$ , the tax rate  $t_t$ , and the benefit generosity  $d_t$ . It faces the flow budget constraint

$$G_t + \sum_{j=0}^J N_{jt} w_t r_{jt} d_t + (1+r_t)B_t = t_t w_t \sum_{j=0}^J N_{jt} (1-r_{jt})^{-j} + B_{t+1}, \quad (A.65)$$

where a positive  $B_t$  denotes government borrowing. In the aggregation, we again use that  $E q e = E q = 1$  for each  $j$  to obtain that average benefits and labor income per age- $j$  person are  $w_t r_{jt} d_t$  and  $w_t (1-r_{jt})^{-j}$  respectively.

The government targets an eventually converging sequence  $\frac{B_{t+1}}{Y_{t+1}} \rightarrow \bar{b}$ . To reach this target, we assume that the government uses a fixed sequence of retirement policies  $r_{jt} = \bar{r}_j$ , and adjusts the other instruments using a fiscal rule defined in term of the "fiscal shortfall"  $S\bar{F}_t$ , defined as

$$\frac{S\bar{F}_t}{Y_t} = \frac{\bar{G}}{Y} + \frac{\sum_{j=0}^T [r_{j,t} \bar{d} - f(1-r_{j,t})^{-j}] N_{jt} w_t}{Y_t} + (r_t - g_t) \frac{B_t}{Y_t} - (1+g_t) \frac{B_{t+1}}{Y_{t+1}} - \frac{B_t}{Y_t}, \quad (A.66)$$

where  $g_t = \frac{Y_{t+1}}{Y_t} - 1$ . The fiscal shortfall is positive at time  $t$  if expenditures minus revenues is too high to reach the debt target when the instruments  $G$ ,  $d$ , and  $t$  are set at some reference levels  $\bar{G}$ ,  $\bar{d}$

and  $\bar{f}$ . Given a non-zero scalar shortfall, the scalar rule consists of three weights  $j^G, j^t, j^d$  and an updating rule for instruments

$$j^G S\bar{f} = (G_t - \bar{G}) \delta t = 0 \quad (\text{A.67})$$

$$j^t S\bar{f} = (t_t - \bar{f}) w_t \sum_{j=0}^J N_{jt} (1 - r_{jt}) \delta t = 0 \quad (\text{A.68})$$

$$j^d S\bar{f} = (d_t - \bar{d}) w_t \sum_{j=0}^J N_{jt} r_{jt} \delta t = 0 \quad (\text{A.69})$$

$$1 = j^G + j^t + j^d, \quad (\text{A.70})$$

where the weights capture the share of the shortfall covered by each instrument.

**Market clearing.** The assets in the economy consist of capital  $K_t$ , government bonds  $B_t$ , and foreign assets  $NFA_t$ . The asset market clearing condition is

$$K_t + B_t + NFA_t = W_t. \quad (\text{A.71})$$

Given the other equilibrium conditions, (A.71) can be used to derive the goods market clearing condition<sup>75</sup>

$$NFA_{t+1} - NFA_t = NX_t + r_t NFA_t + W_{t+1}^{mig}, \quad (\text{A.72})$$

where  $NX_t = Y_t - I_t - C_t - G_t$  is net exports at time  $t$  and

$$W_{t+1}^{mig} = \sum_{j=1}^J M_j \int_{q,e} a_{j,t} p^q(q) p^e(e) \int_a^Z a dH_{j,t+1}(a, q, e) + b_{j,t+1}^q(q)$$

is the assets at time  $t$  that comes from migrants.

**Small open economy equilibrium.** A small open economy equilibrium is defined for:

A sequence of interest rates  $\{r_t\}_{t=0}^\infty$

A government scalar rule  $\{B_{t+1}/Y_{t+1}, r_{jt}, j^G, j^t, j^d, \bar{G}, \bar{f}, \bar{d}\}_{t=0}^\infty$

A sequence of average effective labor supplies  $\{\bar{g}_{jt}\}_{t=0}^\infty, j=1, \dots, J$

An initial distribution of assets  $\{H_{j0}(a|q, e)\}_{j=0}^J$

Technology parameters  $\{Z_0, g, d, n, a, g\}$

Demographics: initial  $\{N_{j,1}\}_{j=0}^J$  and forcing parameters  $\{M_{jt}, f_{jt}, N_{0,t+1}\}_{t=0}^\infty, j=1, \dots, J$

Initial aggregate variables  $K_0, B_0, A_0$

The equilibrium consists of:

Individual decision functions:  $c_{jt}(q, e, a), a_{jt}^0(q, e, a)$

<sup>75</sup>Combine the aggregated household budget constraint with the government budget constraint (A.65), the capital evolution equation (A.63), and the asset market clearing condition (A.71).



A sequence of asset distribution functions  $f H_{jt}(a; q, e) g_{1-t, j}^m$

Government policy variables  $f G_t, t_t, d_t g_t$

A sequence of wages  $f w_t g_t$

A sequence of bequests received  $f b_{jt}(q) g_t$

A sequence of aggregate quantities  $f Y_t, L_t, I_t, K_{t+1}, W_t, C_t, NFA_t g_t$

It is characterized by that

$r_0$  is consistent with  $K_0$   $\Rightarrow$  (A.61) holds given  $K_0$  and  $L_0 = \sum_j N_{j0}(1 - r_{j0})^{-1} j_0$

$W_0$  is consistent with  $H_{j0}$ , that is, (A.58) holds

Individual decision functions solve (A.54).

The set of  $H_{jt}$ 's satisfies the evolution equation (A.55) and the boundary condition (A.56)

The government policy variables satisfy (A.66)-(A.69).

Equations (A.59)-(A.64) hold.

$A_t$  satisfies (A.58) for  $t \geq 0$

$NFA_t = W_t - K_t - B_t$ , with  $B_0$  given by the initial condition, and  $B_{t+1}/Y_{t+1}$  by the government fiscal rule.

Bequests received  $b_{jt}(q)$  satisfy (A.57)

**World-economy equilibrium.** Given a set of countries  $c \in C$ , a world-economy equilibrium is a sequence of returns  $f r_t, f r_t g_{t-1} g$  and a set of corresponding sequences of prices and allocations  $S^c$  for each economy  $c$  such that each  $S^c$  is a small open economy equilibrium, and that their NFAs satisfy

$$\sum_{c \in C} NFA_t^c = 0 \quad \forall t \geq 0 \quad (A.73)$$

## D.2 Proof of proposition 5

Let  $F_t^c$  capture all demographic variables in a country: population shares, fertility, mortality, migration. Given fixed  $r$  and  $B^c/Y^c$ , long-run government policy only depends on  $F^c$ . Wages per unit of effective labor only depend on  $r$ . Assuming that the steady state of the household problem is unique conditional on demographics, wages, and government policy, we can express it as a function of  $(r, F^c)$ .<sup>76</sup> Let  $\frac{W^c}{Y^c}(r, F^c)$  denote the resulting steady-state wealth-to-output ratio.

Output, normalized by technology, only depends on aggregate effective labor supply, which is a function of  $F^c$  (both directly through the number of people at each age and indirectly through

<sup>76</sup>Aside from bequests, we have a standard incomplete markets household problem and this would be a standard result. Bequests introduce some complication, since bequests depend on the endogenous distribution of assets, but household asset policy also depends on realized and expected bequests. The solution to the household problem is a fixed point of this process. We assume that the fixed point is unique and a global attractor; in practice, we have found that this assumption is always satisfied.

government retirement policy), and the capital-to-effective-labor ratio, which is a function of  $r$ . Hence we can write each country's share of global GDP as  $\frac{Y^c}{Y}(r, n, F)$ .

From here on, the proofs of propositions 2 and 3 in appendix B.3 apply, provided that, in equations (A.6) and later, we replace  $p$  with  $F$ ,  $\frac{W^c}{Y^c}(r_0, F_{LR}^c)$  with  $\frac{W^c}{Y^c}(r_0, F_0^c)$  with  $D_{LR}^{soec}$ , as well as  $\frac{W^c}{Y^c}(r_0, F_{LR}^c)$  with  $\frac{W^c}{Y^c}(r_0, F_0^c)$  with  $\bar{D}_{LR}^{soec}$  everywhere.

### D.3 Steady-state equations and calibration details

**Steady-state equations.** Our calibration targets a stationary equilibrium associated with a constant rate of return  $r$ . Most elements are standard: we assume constant technology parameters  $f, g, d, n, a, \bar{g}$ , a constant bond-to-output ratio  $\frac{B}{Y}$ , retirement policy  $r_j$ , tax rate  $t$ , social security generosity  $d$ , and government consumption-to-output ratio  $G/Y$ . We also assume that there is a fixed distribution of assets  $H_j(\tilde{a}|q, e)$ , where  $\tilde{a}$  is assets normalized by technology (again, we drop the country superscripts in the description of each country, and reintroduce them when we define the world equilibrium).

The non-standard element is that we introduce a counterfactual flow of migrants to ensure a time-invariant population distribution and growth rate at their 2016 levels. In particular, demography consists of constant mortality rates, a fixed age distribution, a constant population growth rate, and a constant rate of migration by age  $m_j$ :

$$f_{jt} = f_j, \quad p_{jt} = p_j, \quad N_t = (1+n)^t N_0, \quad m_j = \frac{M_j}{N},$$

and the net migration by age is given by

$$m_{j-1} = \frac{M_{j-1}}{N} = p_j \frac{1+n}{f_{j-1}} - p_{j-1}, \quad (\text{A.74})$$

which ensures that (A.53) holds given a fixed age distribution of population. The notation  $\frac{M_{j-1}}{N}$  without a time index is used to indicate the constant ratio  $\frac{M_{j-1,t}}{N_t}$ . Throughout, we use an analogous notation whenever the ratio of two variables is constant over time.

In normalized form, the consumer problem is

$$\begin{aligned} \tilde{V}_j(q, e, \tilde{a}) = \max_{\tilde{c}, \tilde{a}^0} & \frac{\tilde{c}^{1-\frac{1}{s}}}{1-\frac{1}{s}} + U(1+g)^{1-n-1} f_j \frac{(\tilde{a}^0)^{1-n}}{1-n} + \frac{b_{j+1}}{b_j} (1+g)^{1-\frac{1}{s}} f_j E \tilde{V}_{j+1}(q, e^0, \tilde{a}^0) \\ & c + (1+g)\tilde{a}^0 - \tilde{w}_t q (1-r_j)(1-t)^{-1} e + r_j d + (1+r)\tilde{a} + \tilde{b}_f(q) \\ & \tilde{a} = \tilde{a}^0(1+g), \end{aligned} \quad (\text{A.75})$$

where a variable with a tilde denotes normalization by  $Z_t$ , except for  $\tilde{V}_j = \frac{V_{jt}}{Z_t^{1-\frac{1}{s}}}$ . As elsewhere in the paper, we write  $g$  for the overall growth rate of the economy

$$1+g = (1+n)(1+g).$$

The consumer problem implies decision functions  $\tilde{c}_j(\cdot)$  and  $\tilde{a}_j^0(\cdot)$ , where the latter denotes the choice of next period's normalized assets as a function of the state at age  $j$ . From the evolution

and boundary conditions of assets (A.55) and (A.56), the stationary distribution of assets satisfies

$$H_j(\tilde{a}|q, e) = \begin{cases} \int_{\tilde{a}=0}^{\tilde{a}^e} \frac{P^e(e|e)}{p^e(e)} R_{\tilde{a}^0}^h \tilde{a}_j^0(\tilde{a}^0, q, e) \tilde{a}^i dH_{j-1}(\tilde{a}^0|q, e) & \text{if } j > J^w \\ 0 & \text{if } j = J^w \end{cases}$$

Normalized bequests satisfy

$$p_j \tilde{b}_j^f(q) = F_j \int_{\tilde{a}=0}^{\tilde{a}^e} \frac{P^q(q|q) p^q(q)}{p^q(q)} \int_{k=0}^T [p_k + m_k] \frac{(1 - f_k)}{1 + n} \int_{\tilde{a}=0}^{\tilde{a}^e} p^e(e) \tilde{a} dH_k(\tilde{a}; q, e) \quad (\text{A.76})$$

Aggregate consumption and assets are

$$\frac{C}{NZ} = \int_{j=0}^T \int_{q,e} p_j \int_{\tilde{a}} p^q(q) p^e(e) c_j(\tilde{a}, q, e) dH_j(\tilde{a}, q, e)$$

$$\frac{W}{NZ} = \int_{j=0}^T \int_{q,e} p_j \int_{\tilde{a}} p^q(q) p^e(e) \tilde{a} dH_j(\tilde{a}, q, e) + b_j^f(q)$$

Finally, since we assume that steady state migrants have the same distribution of assets as regular households, we have

$$\frac{A^{mig}}{NZ} = \int_{j=1}^T m_j \int_{q,e} p_j \int_{\tilde{a}} p^q(q) p^e(e) \tilde{a} dH_j(\tilde{a}, q, e) + b_j^f(q) \quad (\text{A.77})$$

where we recall that  $m_j$  is the number of migrants as a share of age group  $j$  at time  $t$ , and  $W_j$  is the total amount of assets of age- $j$  individuals.

The stationary analogues of the production sector equations (A.59)-(A.64) are

$$\frac{Y}{ZN} = F \left( \frac{K}{ZN}, \frac{L}{N} \right) \quad (\text{A.78})$$

$$r + d = F_K \left( \frac{K}{ZN}, \frac{L}{N} \right) = a \left( \frac{K}{Y} \right)^{1/h} \quad (\text{A.79})$$

$$\frac{w}{Z} = F_L \left( \frac{K}{ZN}, \frac{L}{N} \right) \quad (\text{A.80})$$

$$(g + d) \frac{K}{Y} = \frac{I}{Y} \quad (\text{A.81})$$

$$\frac{L}{N} = \int_{j=0}^T p_j (1 - r_j) \quad (\text{A.82})$$

The steady-state government budget constraint is derived from (A.65) given a fixed debt-to-output ratio

$$\frac{G}{Y} + \frac{w}{Y} \frac{d}{Y} \int_{j=0}^T p_j N_j r_j + (r - g) \frac{B}{Y} = t \frac{wL}{Y}, \quad (\text{A.83})$$

and the asset market and goods market clearing conditions are derived from (A.71) and (A.72):

$$\frac{W}{Y} = \frac{K}{Y} + \frac{B}{Y} + \frac{NFA}{Y} \quad (\text{A.84})$$

$$0 = \frac{NX}{Y} + (r - g) \frac{NFA}{Y} + \frac{A^{mig}}{Y(1+g)}. \quad (\text{A.85})$$

The world asset market clearing condition is

$$\sum_c \omega^c \frac{NFA^c}{Y^c} = 0, \quad \omega^c = \frac{Y^c}{\sum_c Y^c} \quad (\text{A.86})$$

## D.4 Calibration details

All demographic data is from the UN World Population Prospects, interpolated across years and ages to obtain data for each combination of year and age. For each country, we use the 2016 values for age-specific survival rates  $f_j^c$  and population shares  $p_j^c$ . The population growth rate is defined as

$$1 + n^c = \frac{N_{2016}^c}{N_{2015}^c}$$

where  $N_{2016}^c$  and  $N_{2015}^c$  are the populations of country  $c$  in 2016 and 2015.

Debt-to-output is from the October 2019 IMF World Economic Outlook, and the net foreign asset position from the IMF Balance of Payments and International Investment Positions Statistics, deflated by nominal GDP from the Penn World Table 9.1.

For each country, the labor-augmenting productivity growth  $g^c$  is defined as the average growth rate between 2000 and 2016 in real GDP divided by effective labor supply. For each country, we measure real GDP as expenditure-side real GDP from the Penn World Table 9.1, effective labor supply as  $L_t^c = \sum_j N_{jt}^c h_j^c$ , with  $N_{jt}^c$  taken from the UN World Population Prospects, and  $h_j^c$  given by the labor income profiles defined in section 3. We define the world  $g$  as the average of  $g^c$ , weighted by real GDP.

Given  $g^c$  and the elasticity of substitution between capital and labor  $\eta$ , the growth rate of each economy is

$$g^c = (1 + n^c)(1 + g^c) - 1,$$

and we calibrate the investment-to-output ratios, the share parameter in the production function, and the labor share

$$\begin{aligned} \frac{I^c}{Y^c} &= \frac{K^c}{Y^c} (d + g^c) \\ a^c &= (r + d) \frac{K^c}{Y^c}^{\frac{1}{\eta}} \\ s^{L,c} &= 1 - (r + d) \frac{K^c}{Y^c}, \end{aligned}$$

where the expression for investment and  $a$  use (A.81) and (A.79). Note that this calibration ensures that the world asset market clearing condition (A.86) holds for  $r$ .

For government policy, we use the average labor wedge from the OECD Social Expenditure Database 2019 to target  $\tau$ . This measure includes both employer and employee social security contribution, which is consistent with treating  $w_t$  as the labor cost for employers. For  $d$ , we use

data on the share of GDP spent on old age benefits, using data on benefits net of taxes from the OECD Social Expenditure Database.<sup>77</sup> Our main source for the retirement age is OECD's data on "Effective Age of Labor Market Exit" from the OECD Pensions at a Glance guide.<sup>78</sup> For some countries, the age provided by the OECD implies that labor market exit happens after the age at which aggregate labor income falls below implied benefit income. In those cases, we define the latter age as the date of labor market exit. Formally, this is done by calibrating the implied benefit levels for each possible retirement age and choosing the highest age at which retirement benefits are weakly lower than net-of-tax income. Last,  $G/Y$  is calibrated residually to target (A.83) given  $B/Y$ ,  $t$ ,  $d$ , and the retirement age.

For individuals, we use Auclert and Rognlie (2018) and De Nardi (2004) to target the standard deviations  $u_e, u_q$  and the persistence parameters  $\rho_e, \rho_q$ . The processes are discretized using Tauchen's method, using three states for  $q$  and 11 states for  $e$ . Both processes are rescaled to ensure that they have a mean of 1.

**Model outcomes and plots.** Figure A.7 and A.8 show the model plots of age profiles of wage and labor income across all countries. For the labor income profile, the orange depicts labor income  $(1 - r_{j0})w_j$  in the initial steady state, and the white hollow dots depict  $w_j$  which become relevant as the retirement age increases.

Table A.5 provides the main parameters for all countries, table A.6 provides additional parameters for all countries. Last, figure A.9 shows the outcomes for bequests and wealth inequality in the US. Panel A compares the distribution of bequests in the model to the empirical distribution in the data. We measure it as the value of bequests at certain percentiles divided by average bequests. We take the empirical distribution from Table 1 in Hurd and Smith (2002). The legend also reports the resulting model aggregate bequests-to-GDP ratio  $\frac{Beq}{Y} = 8.8\%$ . Panel B compares the model Lorenz curve to the one obtained in the SCF. We see that our model produces substantial wealth inequality, with the richest 20% holding roughly 70% of wealth. However, it does not go all the way to match the wealth inequality in the US data.

## D.5 Simulating demographic change

**Solution method.** We solve for the perfect foresight transition path between 2016 ( $t = 0$ ) and 2300 ( $t = 284 - T$ ) as follows.

In every country, we simulate demographics forward using the initial population distribution  $\{N_j, g_{j=0}^J\}$  and the forcing variables  $\{M_{jt}, f_{jt}, N_{0,t+1}g_{1-t,T,0}^J\}$  to obtain  $\{p_{j,t}, N_{j,t}g_{t,T,0}^J\}$  and population growth rates  $\{n_t g_{j=0}^T\}$ . The forcing variables are obtained from the UN World Population Prospects until 2100. From 2100 on, we assume that the survival rates  $f_{jt}$  and migration rates  $\frac{M_{jt}}{N_t}$  are kept constant at their 2100 levels. We further assume that the growth rate of the number of births,  $N_{0,t+1}/N_{0t}$ , adjusts linearly in every country from its 2100 level to a common long-run rate of 0.5% by 2200. Given the effective labor supply profile and the retirement policy, the demographic projections imply a path for aggregate labor  $\{L_t g_{t=0}^T\}$  from (A.64).

Next, given a path for the interest rate  $\{r_t g_{t=0}^T\}$ , technological parameters, and aggregate labor, we can obtain the optimal capital-labor ratio from (A.61) and other production aggregates as well as the wage rate  $\{K_t, Y_t, l_t, w_t g_{t=0}^T\}$  follow from (A.59)-(A.63).

<sup>77</sup>OECD SOCX Manual, 2019 edition.

<sup>78</sup>Pensions at a Glance 2019: OECD and G20 Indicators.

### Figure A.7: Calibration outcomes: wealth

Notes: This figure presents the empirical age-wealth profiles (gray dots) and the calibrated model age-wealth profiles in the baseline calibration (orange line) for the 25 countries we consider.

### Figure A.8: Calibration outcomes: labor income

Notes: This figure presents the empirical age-labor supply profile from LIS used in section 2 (black dots), as well as the model gross age-labor supply profile (dashed orange line) and the net-of-taxes profile (red line).

Table A.5: World economy calibration

Country	$D^{compc}$		Components of wealth			Government policy	
	Model	Data	$\frac{W^c}{Y^c}$	$\frac{B^c}{Y^c}$	$\frac{NFA^c}{Y^c}$	$t^c$	$\frac{Berf}{Y^c}$
AUS	1.72	1.68	5.09	0.40	-0.46	0.29	0.04
AUT	1.14	1.07	3.90	0.83	0.12	0.47	0.11
BEL	1.91	1.85	5.74	1.06	0.65	0.54	0.09
CAN	1.07	1.04	4.63	0.92	0.20	0.31	0.04
CHN	2.50	2.37	4.20	0.44	0.25	0.30	0.04
DEU	0.82	0.79	3.64	0.69	0.58	0.50	0.10
DNK	0.75	0.72	3.42	0.37	0.46	0.36	0.06
ESP	2.77	2.41	5.33	0.99	-0.74	0.39	0.10
EST	0.64	0.63	2.64	0.09	-0.33	0.39	0.07
FIN	0.65	0.66	2.78	0.63	0.16	0.44	0.09
FRA	1.72	1.67	4.85	0.98	-0.05	0.48	0.13
GBR	1.64	1.60	5.35	0.88	0.08	0.31	0.06
GRC	1.56	1.38	4.25	1.81	-1.25	0.40	0.16
HUN	0.49	0.48	2.19	0.76	-0.54	0.48	0.09
IND	3.75	3.12	4.16	0.68	-0.08	0.30	0.01
IRL	1.49	1.46	2.32	0.74	-1.65	0.33	0.03
ITA	2.33	2.06	5.83	1.31	-0.02	0.48	0.13
JPN	1.32	1.19	4.85	2.36	0.66	0.32	0.09
LUX	2.05	1.86	3.92	0.21	0.64	0.40	0.07
NLD	1.57	1.54	3.92	0.62	0.70	0.37	0.05
POL	1.38	1.35	3.31	0.54	-0.52	0.36	0.10
SVK	0.85	0.84	2.17	0.52	-0.59	0.42	0.07
SVN	0.70	0.69	2.82	0.79	-0.21	0.43	0.11
SWE	0.67	0.70	3.81	0.42	0.08	0.43	0.06
USA	1.62	1.47	4.38	1.07	-0.36	0.32	0.06



Table A.6: World economy calibration

Country	$\bar{b}^c$	$x^c$	$U^c$	$n^c$	$a^c$	$G^c/Y^c$
AUS	0.984	0.00022	118.269	1.681	0.500	9.9%
AUT	0.996	-0.00012	118.269	1.681	0.287	22.0%
BEL	0.983	0.00065	118.269	1.681	0.391	22.2%
CAN	1.001	-0.00017	118.269	1.681	0.341	15.5%
CHN	1.024	-0.00003	118.269	1.681	0.341	15.1%
DEU	1.006	-0.00037	118.269	1.681	0.230	27.6%
DNK	1.161	0.00239	118.269	1.681	0.252	20.4%
ESP	0.939	-0.00044	118.269	1.681	0.494	7.6%
EST	1.177	0.00024	118.269	1.681	0.280	21.0%
FIN	1.195	0.00255	118.269	1.681	0.193	25.4%
FRA	1.001	0.00040	118.269	1.681	0.380	15.6%
GBR	1.000	0.00029	118.269	1.681	0.426	10.7%
GRC	1.015	0.00024	118.269	1.681	0.359	6.0%
HUN	1.178	0.00116	118.269	1.681	0.191	28.1%
IND	0.997	0.00041	118.269	1.681	0.347	18.5%
IRL	1.199	0.00284	118.269	1.681	0.314	18.6%
ITA	0.930	-0.00071	118.269	1.681	0.441	11.1%
JPN	1.089	0.00098	118.269	1.681	0.177	12.7%
LUX	1.195	0.00341	118.269	1.681	0.299	20.8%
NLD	1.144	0.00248	118.269	1.681	0.253	21.9%
POL	1.055	0.00057	118.269	1.681	0.319	13.6%
SVK	1.233	0.00199	118.269	1.681	0.218	24.4%
SVN	1.171	0.00076	118.269	1.681	0.219	20.9%
SWE	1.010	-0.00003	118.269	1.681	0.322	23.0%
USA	1.044	0.00063	118.269	1.681	0.356	12.5%

A. US bequests distribution

B. US wealth Lorenz curve

Figure A.9: Distribution of bequests and wealth Lorenz curve in the US

Given a government scal rule  $f_{B_{t+1}/Y_{t+1}, r_{jt}, j^G, j^t, j^d, \bar{G}, \bar{f}, \bar{d}g_{t=0}^T}$ , we obtain the path for the policies  $f_{G_t, t_t, d_t g_{t=0}^T}$  from (A.67)-(A.69) such that the government budget constraint (A.65) is satisfied for every  $t$ .

Then, we solve the household problem as follows. Given a guess for total bequests received by type  $q$  across all ages  $f_{Beq(q)g_{t=0,q}^{79}}$ , a path of prices  $f_{r_t, w_t g_{t=0}^T}$ , government policy  $f_{r_{jt}, t_t, d_t g_{t=0}^T}$ , demographic variables  $f_{n_t, p_{j,t}, f_{j,t} g_{0-t, 0-j}^J}$ , we solve the household problem (A.54) in two steps. First, we use Carroll (2006)'s Endogenous Grid Point Method (EGM) to determine the decision functions  $f_{c_{jt}(q, e, a)g_{t=0,0-j}^T}$  and  $f_{a_{j+1,t+1}(q, e, a)g_{t=0,0-j}^T}$ , assuming constant prices after 2300. Second, we obtain the distributions following Young (2010). We start from an initial distribution, which we take from the 2016 steady-state, and iterate forward using the asset decision function and the law of motion of the state  $(q, e)$ . We then compute aggregates following (A.58).

To solve for the world economy equilibrium, we use a Newton-based method to ensure that bequests received equals bequests given by type  $q$  and that the asset market clearing condition (A.73) is satisfied. We iterate on a 285 1 path for the interest rate by year  $f_{r_t g_t}$ , and a 285 25 3 path for bequest by year, country and type  $f_{B^{r,c}(q)g_{t,c,q}}$  until convergence.

To solve for the small open economy, we hold fixed the path of the interest rate, i.e.  $r_t = r_0, 8t > 0$ .

Details on table 4. Below, we provide details on the results in table 4, starting with the construction of each column, and then the details on the various experiments. The description of the columns applies to the full model analyses; for the pure compositional analysis, some columns have a slightly different interpretation, which is clarified when we discuss this experiment. For all columns, the changes refer to differences between 2016 and 2100. In the left panel  $\bar{D} \log \frac{W}{Y} = \hat{a}_c w^c D_{2100} \log \frac{W^c}{Y^c}$  is the average change in the wealth-to-output ratio, weighted by initial shares of wealth.

In the right panel,  $\bar{D}^{comp} = \hat{a}_c w^c D_{2100}^{c,comp}$  is the average compositional effect between 2016 and 2100, weighted by initial wealth levels. The term  $\bar{D}^{soe} = \hat{a}_c w^c D_{2100}^{c,soe}$  is the equivalent average for the small open economy effect. For each country  $c$ ,  $D^{c,soe}$  is defined as the change in  $\frac{W^c}{Y^c}$  between 2016 and 2100 in a small open economy equilibrium with a fixed interest rate  $r_{2016}$ .

The asset supply and demand semielasticities  $\bar{e}^d = \hat{a}_c w^c e^{c,d}$  and  $\bar{e}^s = \hat{a}_c w^c e^{c,s}$  are the averages of the country semielasticities weighted by initial wealth levels. For each country  $c$ , the asset demand sensitivity  $e^{c,d}$  is defined as the semielasticity of the steady-state  $\frac{W^c}{Y^c}$  with respect to the steady state interest rate.<sup>80</sup> The asset supply sensitivities are given by  $e^{c,s} = \frac{1}{W^c/Y^c} \frac{h}{r+d} \frac{K^c}{Y^c}$ .

The list below describes the pure compositional analysis and the various model experiments. All model experiments feature a retirement age increased by 1 month per year for the first 60 year of the simulation. All start from the steady-state equilibrium calibration.

Pure compositional effect. This row reproduces the exercise in section 3. That is, all changes in  $r$ , wealth, and NFAs are defined using proposition 2 and 3 given the initial wealth weights  $w^c$ , the compositional effects  $D^{comp}$ , and the set of sensitivities  $e^{c,d}$  and  $e^{c,s}$ . The supply sensitivities are given by  $e^{c,s} = \frac{1}{W^c/Y^c} \frac{h}{r+d} \frac{K^c}{Y^c}$ , where  $\frac{K^c}{Y^c}$  is the calibrated capital stock from the steady-state calibration. The demand sensitivities are defined using the expression in

<sup>79</sup> $Beq(q) = \hat{a}_q \frac{P^q(q_j) p^q(q)}{p^q(q)} = \hat{a}_{k=0}^T [N_{k,t-1} + M_{k,t-1}] (1 - f_{k,t-1}) = \hat{a}_e p^e(e) \int_a^R adH_{kt}(ajq, e)$ , so that bequests per age  $j$  person of type  $q$  is  $b_{jt}^r(q) = \frac{F_j}{N_{jt}} Beq(q)$ .

<sup>80</sup>In practice, we calibrate a steady-state to 2100 demographics, and perturb  $r_{2016}$  and resolve for a new stationary equilibrium, using the resulting perturbation to  $\frac{W^c}{Y^c}$  to calculate the derivative.

proposition 4, using the same method as in section 3, but instead using the calibrated profiles of assets and income to back out the consumption profile and calculate the relevant moments of the asset and consumption profiles.

Preferred model specification. The social rule places equal weight on consumption, taxes, and retirement benefits.

Constant bequests. The process  $\frac{b^{jt}(q)}{w_t}$  of bequests received normalized by wages is kept constant over time. This removes a source of non-compositional increases in asset holdings which comes from an older population implying that people receive more bequests over time. To make a constant sequence of bequests consistent with equilibrium, we assume that it is implemented with an age-type specific lump sum tax/transfer that keeps bequests over wages constant at their 2016 level once these additional taxes/transfers are netted out. To prevent this tax from having second order effects on individual behavior through the government budget constraint, we assume that it is neutralized by variation in government consumption.

Constant mortality. The subjective mortality risk of individuals is kept fixed at their 2016 values, while the population evolution still follows the objective mortality risks.

Constant taxes and transfers. The social rule places all weight on adjustments in government consumption, so that taxes and benefits are constant over time.

Constant retirement age. The retirement age is kept fixed at its 2016 level.

No income risk. The idiosyncratic income risk is switched off and the model is recalibrated.

Annuities. Households get access to annuities, the bequest preference is set to zero  $\beta = 0$ , and the model is recalibrated.

Fiscal rules. The full adjustment weight is placed on either  $G$ ,  $d$ , or  $t$ .

Changes to net foreign asset positions. Appendix Figure A.10 summarizes the model's predictions for the change in net foreign asset position in each country from 2016–2100. Panel A compares the full model findings to the method used in section 3 by plotting the full model results on the vertical axis, and the prediction based on demeaned compositional effects  $D^{\text{comp}} - \bar{D}^{\text{comp}}$  on the horizontal axis. The compositional predictions are generally quite accurate, and the line of best fit excluding India is close to 45 degrees. In India, however, the model predicts even larger net foreign asset position growth than expected from the compositional effect.

Panel B shows that this discrepancy disappears, and the fit is even closer, when we use the demeaned small open economy effect  $D^{\text{soec}}$  for predictions on the horizontal axis instead. This shows that discrepancies in panel A, including for India, are mostly due to the non-compositional effects  $D^{\text{soec}} - D^{\text{comp}}$  of aging in our model, rather than non-linearities or heterogeneity in elasticities.

A. Model DNFA/ Y vs. demeaned  $D^{\text{comp}}$

B. Model DNFA/ Y vs. demeaned  $D^{\text{soe}}$

Figure A.10: Predicting change in net foreign asset position

Notes: Panel A presents the model-implied change in NFA/ Y between 2016 and 2100 on the y-axis, and on the x-axis the change in NFA/ Y predicted from the demeaned model compositional effect,  $\text{NFA/ Y} \exp(D^{\text{comp}} - \bar{D}^{\text{comp}}) - 1$ , over the same period. The dotted line is a 45 deg line. The dashed line is a regression line, and the solid line is this same regression line when India is excluded. Panel B also shows the model DNFA/ Y on the y-axis, but the x-axis presents the change in NFA/ Y predicted from the demeaned model small open economy effect,  $\text{NFA/ Y} \exp(D^{\text{soe}} - \bar{D}^{\text{soe}}) - 1$ .

## E Appendix to Section 5

We first prove the results in the main text. Defining savings for an individual of age  $j$  in state  $(z^j, a_t)$  at time  $t$  as

$$s_t = r a_t + w_t (1 - t)'(z_j) + \text{tr}(z^j) - c_t$$

and using the budget constraint (1), we see that aggregate savings for agents of age  $j$  is given by

$$s_{jt} = E s_t = f_j a_{j+1,t+1} - a_{jt} \quad (\text{A.87})$$

Next, since lemma 1 implies  $a_{jt} = a_j(r) Z_t$ , we have

$$s_{jt} = f_j (1 + g) a_{j+1} - a_j(r) Z_t = s_j(r) Z_t$$

Hence, defining aggregate savings as

$$S_t = \hat{a} N_{jt} s_{jt} \quad (\text{A.88})$$

we have that

$$\frac{S_t}{N_t} = \hat{a} p_{jt} s_{jt} = \hat{a} p_{jt} s_j(r) \underbrace{Z_0}_{s_0} (1 + g)^t = \hat{a} p_{jt} s_0 (1 + g)^t$$

Taking the ratio of this expression to equation (8), we obtain the equivalent of Proposition 1,

$$\frac{S_t}{Y_t} = \frac{F_L(k(r), 1)}{F(k(r), 1)} \frac{\hat{a} p_{jt} s_0}{\hat{a} p_{jt} h_{j0}} \quad (\text{A.89})$$

which delivers equation (28).

Next, combining (A.87), (A.88), and the population dynamics equation  $N_{j+1,t+1} = f_j N_{jt}$ , we have

$$S_t \dot{\Delta} N_{jt} S_{jt} = \dot{\Delta} N_{jt} f_j a_{j+1,t+1} \dot{\Delta} N_{jt} a_{jt} = \dot{\Delta} N_{j+1,t+1} a_{j+1,t+1} \dot{\Delta} N_{jt} a_{jt} = W_{t+1} - W_t$$

where the last line uses the initial and terminal condition on wealth by age. Hence, the aggregate savings rate is:

$$\frac{S_t}{Y_t} = \frac{W_{t+1} - W_t}{Y_t} = \frac{Y_{t+1} W_{t+1}}{Y_t Y_{t+1}} - \frac{W_t}{Y_t} = (1 + g_{t+1}) \frac{W_{t+1}}{Y_{t+1}} - \frac{W_t}{Y_t}$$

where  $g_t$  is the growth rate of aggregate GDP, the sum of productivity growth, population growth and changing composition,

$$1 + g_{t+1} = \frac{Y_{t+1}}{Y_t} = (1 + g) \frac{N_{t+1} \dot{\Delta}_j p_{j,t+1} h_{j0}}{N_t \dot{\Delta}_j p_{jt} h_{j0}} = (1 + g) (1 + n_{t+1}) \frac{\dot{\Delta}_j p_{j,t+1} h_{j0}}{\dot{\Delta}_j p_{jt} h_{j0}}$$

In steady state, therefore, we have

$$\frac{S}{Y} = g \frac{W}{Y}$$

where  $1 + g = (1 + g) (1 + n)$ . This is the famous [Solow \(1956\)](#)–[Piketty and Zucman \(2014\)](#) formula for the relationship between the net savings rate  $W/Y$ , the growth rate of GDP  $g$ , and the wealth-to-GDP ratio  $W/Y$ .

Finally, towards our implementation, we show that  $S_t/Y_t$  can be calculated from the cross-sectional profiles of assets  $a_{jt}$  and demographic projections alone. We first show that  $S_t/Y_t$  in equation (28) can be calculated from cross-sectional age profiles of assets  $a_{j,0}$ . Indeed, we have, starting from  $S_t = W_{t+1} - W_t$ , we have

$$\begin{aligned} \frac{S_t}{N_t (1 + g)^t} &= \frac{W_{t+1}}{N_t (1 + g)^t} \dot{\Delta}_j p_{jt} a_{j0} \\ &= (1 + n_{t+1}) (1 + g) \dot{\Delta}_j p_{j,t+1} a_{j0} - \dot{\Delta}_j p_{jt} a_{j0} \\ &= ((1 + n_{t+1}) (1 + g) - 1) \dot{\Delta}_j p_{jt} a_{j0} + (1 + n_{t+1}) (1 + g) \dot{\Delta}_j p_{j,t+1} - \dot{\Delta}_j p_{jt} a_{j0} \\ &= g_{t+1}^{ZN} \dot{\Delta}_j p_{jt} a_{j0} + (1 + g_{t+1}^{ZN}) \dot{\Delta}_j D p_{j,t+1} a_{j0} \end{aligned}$$

where we have defined  $1 + g_{t+1}^{ZN} = (1 + n_{t+1}) (1 + g)$ . Taking the ratio of this expression to equation (8), we have the following expression for the aggregate savings rate:

$$\frac{S_t}{Y_t} = \frac{F_L(k(r), 1)}{F(k(r), 1)} \frac{g_{t+1}^{ZN} \dot{\Delta}_j p_{jt} a_{j0} + (1 + g_{t+1}^{ZN}) \dot{\Delta}_j D p_{j,t+1} a_{j0}}{\dot{\Delta}_j p_{jt} h_{j0}} \quad (A.90)$$

which is an alternative to equation (A.89).

In principle, to project savings rates from demographic composition, we could equally well implement equation (A.89) or equation (A.90). [Summers and Carroll \(1987\)](#), [Auerbach and Kotlikoff \(1990\)](#), and [Bosworth et al. \(1991\)](#) follow the first route. We prefer to follow the second because it only requires only information that we have already used so far in the paper, and because the computation of age-specific savings rates is subject to a large amount of measurement error.

Figure A.11: Predicted change in savings-to-GDP from compositional effects

Notes: This figure depicts the evolution of the predicted change in the savings-to-GDP ratio from the compositional effect for  $t = 1950$  to  $2100$ , reported in percentage points. The base year is  $2016$  (vertical line). The solid orange line corresponds to the medium fertility scenario from the UN, the dashed green line to the low fertility scenario, and the dashed red line to the high fertility scenario.

## F Interpreting literature findings

In this appendix, we show that our results are useful to understand existing findings in the literature. First, across papers that conduct a similar exercise, we trace results back to their inputs, and show why different assumptions about the compositional effect are a critical driver of the differences in general equilibrium outcomes. Second, within papers that consider the role of parameter changes, we show that our results are useful in explaining the functional form relationship between these parameters and general equilibrium outcomes. In the interest of space, we focus on the effect of demographic change on the total return  $r$  (sometimes referred to as the natural interest rate, or  $r_n$ , in the literature).

### F.1 Explaining different magnitudes across papers

Eggertsson et al. (2019) (EMR) and Gagnon et al. (2021) (GJLS) are two recent papers that find very different effects of demographics on real interest rates. Both study the US economy using closed-economy general equilibrium models, but EMR find that demography reduced real interest rates by 3.44 percentage points between 1970 and 2015, while GJLS only find an effect of 0.92 percentage points, a difference of 2.52 percentage points. We use publicly available replication files<sup>81</sup> to create table A.7, which applies the framework of proposition 5 to explain these results in terms of the underlying differences in compositional effects  $D^{\text{comp}}$ , non-compositional effects  $D^{\text{soe}} - D^{\text{comp}}$ , and semielasticities  $e^d$  and  $e^s$ .

The single most important difference is that the compositional effect in EMR is more than three times as large as that in GJLS. If EMR had the same compositional effect as GJLS, more than half of the gap between the two estimates would be closed. EMR also have a far lower asset supply semielasticity  $e^s$ , one-fourth as large as GJLS. If EMR also had the same  $e^s$  as GJLS, 86% of the gap would be closed.<sup>82</sup>

The results on compositional effects can be interpreted using figure A.12, which shows the asset profiles by age and the population distribution shifts in the two papers and in the data. Two forces explain the large compositional effect in EMR. First, the age-wealth profile is much steeper than in the data, staying below zero until age 46 and then rising sharply. This intensifies the effect of shifting the age distribution toward older ages. Second, the shift in age composition itself is very large, because the exercise compares a steady state based on 2015 fertility and mortality with a steady state based on 1970 demographics (for which EMR take 1970 mortality and, since agents in the model come of age after 25 years, 1945 fertility). Due to the slow convergence rate of the empirical age distribution, these two steady states have larger differences in age distribution than the actual change that occurred between 1970 and 2015.<sup>83</sup>

<sup>81</sup>Replication repositories: <https://www.openicpsr.org/openicpsr/project/114159/version/V1/view> (EMR) and <https://sites.google.com/site/etigag/gjls-replication-materials> (GJLS).

<sup>82</sup>The difference in  $e^d$  in table A.7 also appears substantial, at 12.7 in EMR vs. 28.5 in GJLS. However, the asset demand curve exhibits some non-linearity in response to EMR's very large change in  $r$ , so that if  $e^d$  is taken around the 1970 steady state instead, it is 19.7, considerably closer to GJLS. If we move toward a second-order approximation by taking the average  $e^d = (12.7 + 19.7)/2 = 16.2$ , then  $D^r$  in table A.7 becomes an extremely accurate approximation, at 3.5%. With this  $e^d$ , the compositional effect and  $e^s$  together explain 87% of the difference between the two papers.

<sup>83</sup>In addition to this comparison of steady states, EMR also perform an exercise with explicit transitional dynamics. This exercise features a smaller  $D^{\text{comp}}$  for 1970 to 2015—albeit one that is still somewhat overstated, due to the steep age-wealth profile and since the exercise starts with the 1970 steady state. Overall, however, the decline in  $r$  in this exercise from 1970 to 2015 is quite similar to the decline in  $r$  in the steady

Table A.7: Decomposing the change in equilibrium  $r$  in existing papers

	Eggertsson et al. (2019)	Gagnon et al. (2021)	Sufficient statistic
Time-period	1970–2015	1970–2015	1970–2015
GE transition			
$Dr^{GE}$	3.44%	0.92%	
First-order approximation $Dr = \frac{D^{soe}}{e^d + e^s}$			
$Dr$	4.30%	0.97%	0.49%
$D^{comp}$	45.4%	13.4%	12.4%
$D^{soe} \quad D^{comp}$	21.1%	25.3%	0%
$e^s$	2.8	11.1	8.0
$e^d$	12.7	28.5	17.5
$s$	0.75	0.5	0.5
$h$	0.6	1.0	1.0

Notes: This table analyzes two key results from Eggertsson et al. (2019) (EMR) and Gagnon et al. (2021) (GJLS) using the framework of proposition 5. In GJLS, we analyze the 1970 to 2015 segment of the paper's main experiment, which is a simulation of the effects of demographic change between 1900 and 2030. In EMR, we analyze jointly the two demographic experiments from table 6 ("mortality rate" and "total fertility rate"). These are steady state experiments that consider the effect of changing fertility and mortality from their 2015 to their 1970 level. For both experiments,  $Dr^{GE}$  is the general equilibrium change in  $r$  from 1970 to 2015,  $D^{comp}$  is our compositional effect measure, implemented using the two papers' 2015 age profiles and the age distributions for 1970 and 2015, and  $e^s$  is the semielasticity of asset supply  $(B + K) / W$  in 2015 with respect to  $r$ . For EMR,  $D^{soe}$  is given by the change in  $W / Y$  between the 1970 and 2015 steady state when both have  $r = r_{2015}$  and  $e^d$  is the derivative of  $\log W / Y$  to  $r$  in the 2015 steady state. For GJLS  $D^{soe}$  is the counterfactual change in  $W / Y$  in a simulation where  $r$  is fixed after 1970, and  $e^d$  is the derivative of  $\log W / Y$  to  $r$  around a steady state defined to have the same population age distribution as the one observed in 2015. The sufficient statistic column applies the method in section 3 to 1970-2015, constructing  $D^{comp}$  from observed changes in the age distribution from 1970 to 2015 together with age profiles of assets and labor income from 2016, and asset semielasticities from (23) and proposition 4, for  $e^s$  using the 2016 value of  $K / W$ , and for  $e^d$  using the 2016 profiles of assets and labor income, together with  $s = 0.5$  and  $h = 1$ .

For the asset supply semielasticity  $e^s$ , the lower value in EMR partly reflects their assumption of a lower elasticity of substitution between capital and labor relative to GJLS ( $h = 0.6$  versus  $h = 1$ ). However, even with  $h = 1$ , EMR would only have  $e^s = 4.6$ , less than half that of GJLS. The remaining difference reflects a second, more subtle, reason for EMR having a low  $e^s$ , namely that  $e^s$  scales with the share of capital in total wealth  $K / W$ , which is 1 in GJLS and only 0.51 in EMR. Capital is a small part of wealth in EMR because high (uncapitalized) markups mean that capital owners only receives 10% of total output, with a resulting low capital-output ratio of  $K / Y = 124\%$ . Combined with a high level of bonds  $B / Y = 117\%$ , capital becomes a small part of total wealth, lowering the responsiveness of asset supply to changes in  $r$ .

state exercise. This is for a reason we saw in figure 8. In equilibrium,  $r$  tends to overshoot what current demographics would imply, incorporating future demographic change as well; if  $r$  is only allowed to vary from its initial steady state starting in 1970, as in this exercise (but not GJLS), much of the effect of long-run demographic change is compressed into the 1970–2015 period. Because of this difficulty in interpretation, and because the steady-state exercise is the only one for which EMR explicitly do a breakdown into demographic causes of the decline in  $r$ , we focus on the steady-state exercise in table A.7.



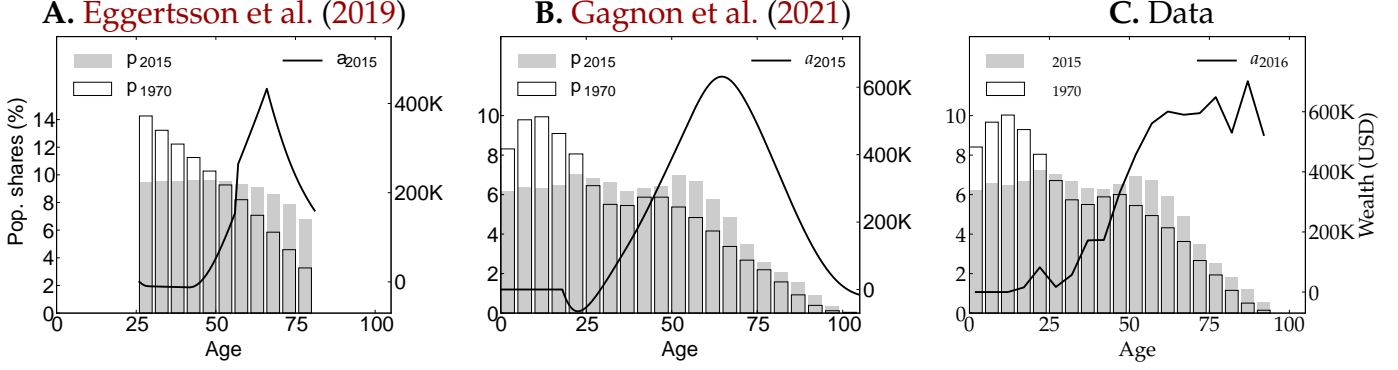


Figure A.12: Age-wealth profiles in papers vs the data

For comparison, we also include the results of the sufficient statistic analysis from section 3 applied to the same time period. For  $D^{comp}$ , the sufficient statistic result comes directly from the data and is closer to GJLS than to EMR. This reflects the fact that GJLS closely target the change in age distribution over time, and also do a good job fitting the age profile of wealth for all but the highest ages, which are of limited quantitative importance before 2015. For  $e^s$ , the results in the sufficient statistic analysis lie above EMR and below GJLS. Apart from having a higher  $h$  than EMR, this mainly reflects the fact that our assumed share of capital in wealth  $K/W = 0.76$  is between the values in GJLS and EMR.

While the non-compositional effects  $D^{soe} - D^{comp}$  are zero in the sufficient statistic analysis, they are positive in EMR (21.1%) and GJLS (25.3%), and relatively large compared to what we find in the quantitative analysis in section 4. The non-compositional effect is especially pronounced in GJLS, where it is twice as large as the compositional effect. This reflects a very strong response of asset accumulation to falling mortality. This is largely due to the lack of bequest motive in GJLS, which implies that all saving is for personal consumption needs, which scale proportionally with survival probabilities. In our model in section 4, the bequest motive scales with mortality and counterbalances this effect; the role of saving for personal consumption in retirement is further diluted by the presence of a social security system.

## F.2 Understanding the role of parameter changes

Our results in section 2 uncover a structural relationship between primitive parameters, calibration moments, and general equilibrium counterfactuals. For instance, combining the results in equations (13) and (17), the inverse effect on the interest rate of a change in demographics that creates a compositional effect of  $\bar{D}^{comp}$  is given by a simple affine function,

$$\frac{1}{dr} = -\frac{\bar{e}^{income} - \bar{e}^{laborshare}}{\bar{D}^{comp}} - S \frac{\bar{e}^{substitution}}{\bar{D}^{comp}} - h \frac{\bar{e}^{laborshare} + \frac{1}{r+d} \frac{\bar{K}}{\bar{W}}}{\bar{D}^{comp}} \quad (\text{A.91})$$

Plugging in the elasticity values from section 3.2, we obtain

$$\frac{1}{dr} = \frac{7.5}{\bar{D}^{comp}} - S \frac{39.5}{\bar{D}^{comp}} - h \frac{13.5}{\bar{D}^{comp}}$$

**Table A.8:** Understanding the functional form relationship between  $s$ ,  $h$  and  $dr$ 

$1/s$	$s$	$dr$ for $h = 1$	$1/dr$	$h$	$dr$ for $1/s = 2.5$	$1/dr$
1	1.00	-0.41	-2.42	0.4	-1.70	-0.59
1.5	0.67	-0.65	-1.54	0.6	-1.44	-0.69
2	0.50	-0.84	-1.19	0.8	-1.20	-0.83
2.5	0.40	-1.00	-1.00	1	-1.00	-1.00
3	0.33	-1.14	-0.88	1.2	-0.84	-1.18
3.5	0.29	-1.25	-0.80	1.4	-0.73	-1.37
4	0.25	-1.35	-0.74	1.6	-0.66	-1.52

Notes: This table presents Papetti (2019)'s findings for the equilibrium change in the real interest rate between 1990 and 2030 ( $dr$ ) as a function of risk aversion  $1/s$  and capital-labor substitution  $h$ . The numbers are taken from his Figures 10 and 12, and then transformed to make the additively linear relationship between  $1/dr$  and  $s$  and  $h$ , which is implied by our framework, appear.

For the 2016-2100 period, we can take  $\bar{D}^{comp} = 32\%$  from section 3, and obtain (for  $r$  in %)

$$\frac{1}{dr} = 0.23 - 1.23 \cdot s - 0.42 \cdot h$$

Equation (A.91) shows that, conditional on having recalibrated the model to hit the same data moments and therefore the same  $\bar{D}^{comp}$  and  $\bar{e}'s$ ,<sup>84</sup> the effects of  $s$  and  $h$  are additively separable for the inverse general equilibrium effect on interest rates,  $1/dr$ .

To illustrate the potential of this equation for interpreting findings in other papers, we study the results in Papetti (2019), who provides a comprehensive structural OLG quantitative model of the Euro Area. In Figures 10 and 12, the author reports his model's predicted effect of demographics on the change in the real interest rate change over the period 1990 – 2030, which we call  $dr$ , first as a function of risk aversion  $1/s$ , and then as a function of capital-labor substitution  $h$ . We reproduce his results in table A.8. Observe that all his estimates of the effect of demographics on interest rates over this period are all negative.

Note further that the inverse effect on the interest rate,  $1/dr$ , appears to be linear in both  $s$  and  $h$ , just like equation (A.91) predicts. To confirm this, we run a linear regression of  $1/dr$  on  $s$  and  $h$  and obtain:

$$\frac{1}{dr} = 0.67 - 2.22 \cdot s - 0.81 \cdot h$$

with an  $R^2$  of 0.993. The quality of the fit of the functional form is remarkable. The coefficients are around two times larger than our coefficient for 2016-2100, so the interest rate effects are about half in our model what they are in his. One obvious distinction is that our results are for an 80 year period, while his are for a 40 year period. In addition, the fundamental inputs into  $e^{substitution}$ ,  $e^{income}$  are different, and the compositional effects  $\bar{D}^{comp}$  in his model appear to be lower than in ours, perhaps because Papetti (2019) does not directly target wealth profiles in his calibration.

<sup>84</sup>In practice, changing  $h$  does not change the steady state so does not require a recalibration, while changing  $s$  requires adjusting parameters to keep  $r$  and the age profiles of assets unchanged.